International Journal of Recent Engineering Research and Development (IJRERD)
ISSN: 2455-8761
www.ijrerd.com || Volume 08 – Issue 05 || May 2023 || PP. 36-43

Optimum tilt angle calculation of a solar panel for maximizing the energy output

Megha Jasmin Benny[1*], Sandhya P Gopal[2]
[1] *M Tech Energy Management Student, Department of Mechanical Engineering,*
Model Engineering College Thrikkakara, Kochi - 682021, India
[2]*Assistant Professor, Department of Mechanical Engineering,*
Model Engineering College Thrikkakara, Kochi - 682021, India
[*]*Email Address: meghabenny107@gmail.com*

Abstract: A solar collector is required to absorb radiation from the sun, and the absorbed energy is transferred into a heat transfer fluid with a minimum of heat loss. The determination of tilt angle is used for maximising the solar energy which falls on the solar panels. Since the angle of the panel is site-specific, it depends on monthly, seasonal and yearly optimum tilt angles. Various methods are used to determine the optimum tilt angle for different locations. The equations are solved theoretically, and a program is developed in "C" to calculate the optimum tilt angle for monthly, seasonally and yearly for the cities of Trivandrum (8.5241°N, 76.9366°E), Kochi (9.9312°N, 76.2673°E), Kozhikode (11.2588°N, 75.7804°E), Chennai (13.0827°N, 80.2707°E), Mumbai (19.0760°N, 72.8777°E) and Delhi (28.7041°N, 77.1025°E) where the energy can be maximised. It has been inferred that the optimum tilt angle increases with the increase in the latitude.

Keywords: Solar radiation, Optimum angle, Maximum Energy, Solar collector.

1. Introduction

Solar radiation is received at the earth's surface in an attenuated form because it is subjected to mechanisms of absorption and scattering as it passes through the earth's atmosphere. Scattering occurs due to the interaction of radiation with particulate matter present in the atmosphere. The scattered radiation is redistributed in all directions, some reflected back into space and some reaching the Earth's surface. Solar Radiation received at the surface without change of direction, i.e. in line with the sun, is called beam radiation or direct radiation. The radiation received at the earth's surface from all parts of the sky's hemisphere (after being subjected to scattering in the atmosphere) is called diffuse radiation. The sum of beam or direct radiation and diffuse radiation is mentioned as total or global radiation.

The intensity of diffuse radiation coming from various directions in the sky is not uniform. The diffuse radiation is said to be anisotropic in nature. However, in many situations (e.g. part or heavy cloud cover), the intensity from all directions tends to be reasonably uniform. It is then modelled as being perfectly uniform and is said to be anisotropic in nature.

The design of devices like concentrating collectors of solar thermal applications or the determination of the area of a solar PV array at a specified location requires knowledge of a variation with time of direct and/or global radiation at the location. In the past, designers of solar equipment resorted to one of the following options:

1. Making radiation measurements over a period of time at the location in question where the solar equipment is to be installed
2. Using measurements available for some other location where the climate is known to be reasonably similar to the location under consideration.
3. Using empirical predictive equations which link solar radiation values with other meteorological parameters whose values are known for the location under consideration. Such parameters include the number of sunshine hours received per day and the cloud cover etc.

Consequently, several models have been developed and are increasingly used for calculating the availability of global/beam radiation at a specified location.

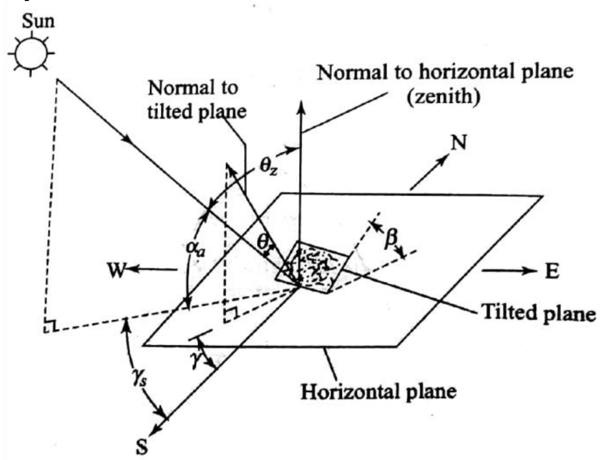
The project's scope is to calculate the optimum tilt angle of a solar panel where the energy output can be maximised by varying the tilt angle monthly, seasonally and yearly for a specified location.

2. Literature Review

The solar panels which receive the radiation get attenuated from the earth's atmosphere in a scattered form. We know that the solar panel is to be kept at a specific position or tilt angle in order to receive maximum radiation. Various Papers have provided various methods for obtaining the optimum tilt angle in order to obtain the maximum radiation. In this paper, the theoretical aspect has been considered as a method to obtain the optimum angle, which will be useful for domestic or industrial purposes by keeping them at a position where the energy output of the solar panel can be maximised for that specific location. Solar panels extract maximum radiation according to the panel faced to the south at a specific tilt angle[1]. The angle of the panel is site-specific and depends upon the sun's yearly, monthly and daily path. The correlations developed to obtain solar radiation were taken from the book on solar energy [2] [3]. Cooper [4] has given a simple relation for calculating the declination angle. Its accuracy of prediction is adequate for engineering purposes. The calculation of Ho has been simplified by Klein [5], who has determined the particular day in each month on which extra-terrestrial radiation is nearly equal to the monthly mean value. The dates on which the value of Ho is equal to are as follows: January 17, February 16, March 16, April 15, May 15, June 11, July 17, August 16, September 15, October 15, November 14 and December 10. As expected, these days are close to the middle of the month. The values of a and b have been obtained by regression analysis of measured values of global solar radiation and sunshine duration of many cities in the world. In India, set values have been obtained for 17 cities [6] . Garg and Garg [7] have examined radiation data for 11 Indian cities and proposed an equation and equation that predicts values for diffuse radiation. This is due to the fact that the diffuse component is, in fact, much larger in India. The daily radiation falling on a tiled surface is also interesting in many applications. Liu and Jordan [8] have proposed the ratio of daily radiation falling on such a surface to the daily global radiation on a horizontal surface. The concept of determining the optimum tilt angle was adopted from the paper of Jamil Ahmed and Tiwari [9].

3. Methodology

3.1 Solar Radiation Geometry



The angle θ can be related by a general equation to φ to the latitude, β the slope, γ the surface azimuth angle, the declination angle and ω the hour angle. Each of these will first be defined:

Latitude (φ)- It is the angle made by the radial line joining the location to the center of the earth with projection of the line on the equatorial plane. By convention, the latitude is measured as positive for the northern hemisphere. It can vary from -90^o to $+90^o$.

Slope (β) - It is the angle made by the plane surface with the horizontal. It can vary from 0^o to 180^o.

Surface Azimuth Angle (γ) - It is the angle made in the horizontal plane between the horizontal surface line due south and the projection of the normal to the surface on the horizontal plane. It can vary from -180^o to $+180^o$. The convention here adopted is to take the angle to be positive if the normal is east of south and negative if west of south.

Declination Angle (δ)is the angle made by the line joining the centers of the sun and the earth with the projection of this line on the equatorial plane.

$$\delta = 23.45 \sin(\frac{360}{365}(284 + n)) \quad (3.1)$$

Where n is the day of the year, δ is expressed in degrees(^{o})

Hour Angle (ω) - It is the angular measure of time and is calculated by the earth's daily rotation around an axis. It is equivalent to 15^o per hour. It also varies from -180^o to $+180^o$. The convention is adopted to measure it from noon based on local apparent time (LAT). Further, it is taken to be positive in the morning and negative in the afternoon

$$\omega = 15(LAT - 12) \quad (3.2)$$

Sunrise/Sunset hour angle for horizontal surface (ω_s) - The hour angle corresponding to sunrise or sunset (ω_s) on a horizontal can be found using the equation:

$$\pm \mid \omega_s \mid = \arccos(-\tan\phi \tan\delta) \quad (3.3)$$

The above equation yields a positive and negative value for ωs, the positive value corresponding to sunrise and the negative to sunset. Since 15o of the hour angle is equivalent to 1 hour, the corresponding day length (in hours)

$$\bar{N} = \frac{2}{15} * \omega_s \quad (3.4)$$

Sunrise/Sunset hour angle for tilted surface (ω_{st}) - The hour angle corresponding to sunrise or sunset (ω_{st}) on a tilted can be found using the equation:

$$\pm \mid \omega_{st} \mid = min[\arccos(-\tan(\phi - \beta)\tan\delta), \arccos(-\tan\phi\tan\delta)] \quad (3.5)$$

Monthly average daily extraterrestrial radiation on a horizontal surface (Ho) - The extra-terrestrial radiation is defined as the radiation that passes perpendicularly through the imaginary surface just outside the earth's atmosphere. It varies from day to day, depending on the distance between the sun and the Earth. It is given by:

$$\bar{H}_o = \frac{24}{\pi} G_{sc} * 3600(1 + 0.033\cos\frac{360n}{365})(\cos\phi\cos\delta\sin\omega_s + \omega_s\sin\phi\sin\delta) \quad (3.6)$$

H o = kJ/m^2
G_{sc} = 1.367 kW/m^2 (solar constant)

Monthly average daily global radiation on a horizontal surface (Hg)- The monthly average daily global radiation is given by:

$$\frac{\bar{H}_g}{\bar{H}_o} = a + b(\frac{\bar{n}}{\bar{N}}) \quad (3.7)$$

H o= Monthly average daily extraterrestrial radiation on a horizontal surface (kJ/m^2)
$\bar{n}$= Monthly average daily hours of bright sunshine (hrs).
N = Monthly average of maximum possible sunshine hours per day, i.e. the day length on a horizontal surface (hrs).
a and b = constants obtained by fitting data.

Monthly average daily diffuse radiation on a horizontal surface (H_d) - Garg and Garg have examined the radiation data for 11 Indian cities and pro- posed this equation and is given by:

$$\frac{\bar{H}_d}{\bar{H}_g} = 0.8677 - 0.7365(\frac{\bar{n}}{\bar{N}}) \quad (3.8)$$

H_g=Monthly average daily global radiation on a horizontal surface (kJ/m^2)
$\bar{n}$=Monthly average daily hours of bright sunshine (hrs).
N =Monthly average of maximum possible sunshine hours per day, i.e. the day length on a horizontal surface (hrs).

Monthly average daily beam radiation on a horizontal surface (H_b) - The monthly average daily beam radiation can be found using the equation given below:

$$\bar{H}_b = \bar{H}_g - \bar{H}_d \quad (3.9)$$

H_g=Monthly average daily global radiation on a horizontal surface (kJ/m^2).
H_d=Monthly average daily diffuse radiation on a horizontal surface (kJ/m^2).

Monthly average daily total radiation on a tilted surface (H_T) - The daily radiation falling on a tilted surface is also of interest in many applications. Liu and Jordan have proposed that the ratio of daily radiation falling on tilted surface (H_T) to the daily global radiation on a horizontal surface(H_g) is given by an equation below:

$$\frac{\bar{H}_T}{\bar{H}_g} = (1 - \frac{\bar{H}_d}{\bar{H}_g})R_b + (\frac{\bar{H}_d}{\bar{H}_g})R_d + R_r \qquad (3.10)$$

The ratio of beam radiation on a tilted surface to that on a horizontal surface (R_b) and is given by the equation:

$$R_b = \frac{\omega_{st} \sin\delta \sin(\phi - \beta) + \cos\delta \cos(\phi - \beta) \sin\omega_{st}}{\omega_s \sin\phi \sin\delta + \cos\phi \cos\delta \sin\omega_s} \qquad (3.11)$$

The ratio of diffuse radiation on a tilted surface to that on a horizontal surface (R_d). The value of this tilt factor depends upon the distribution of diffuse radiation over the sky on the portion of the sky dome seen by the tilted surface. Assuming that the sky in isotropic source of diffuse radiation, the equation is given by:

$$R_d = \frac{1 + \cos\beta}{2} \qquad (3.12)$$

The ratio of reflected radiation on a tilted surface to a horizontal surface(Rr). Assuming that the reflection of the beam and diffuse radiations falling on the ground is diffuse and isotropic and that the reflectivity is ρ, the tilt factor for reflected radiation is given by;

$$R_r = \rho(\frac{1 - \cos\beta}{2}) \qquad (3.13)$$

Where solar reflectivity ρ is assumed to be taken as 0.2.

Using the above equations (3.1-3.13), the total radiation falling on a tilted surface by varying the tilt angles from 0^o to 90^o with a 10^o interval for each month of the year for 6 stations. Graphs have been plotted between total radiation falling on the tilted surface and the tilt angle for each month of the year. With the help of the graph, second-degree polynomials were obtained for curve fitting in the plots. These equations were differentiated with respect to the tilt angles and equated to zero in order to obtain the optimum tilt angle according to the maximum radiation falling on the tilted surface for each month of the year. A program in ”C” has been developed using the above equations to calculate the radiation falling on the tilted surface by varying the tilt angle from 0^o to 90^o with a 10^o interval.

4. Results & Discussions

4.1 Results

To maximise the energy output from the solar panels, the panels should be pointed in a direction that captures maximum energy from the sun. If we are in the northern hemisphere, the panels should be faced true south, and if in the Southern hemisphere should be faced north. In this paper we discussed determining the tilt angle for monthly, seasonal and yearly, where we can maximise the panel’s energy output.

Here is the list of typical days per month, which has been proposed by Klein.

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Days Typical	17	16	16	15	15	11	17	16	15	15	14	10
No.of Days	17	47	75	105	135	162	198	228	258	288	318	344

Table 4.1: List of Typical days

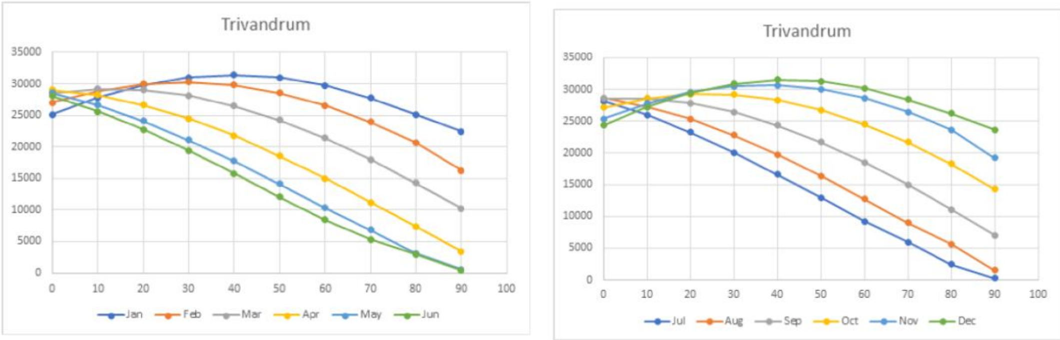


Fig 4.1: Monthly average daily total solar radiation on a tilted surface for the months of January-December (Trivandrum)

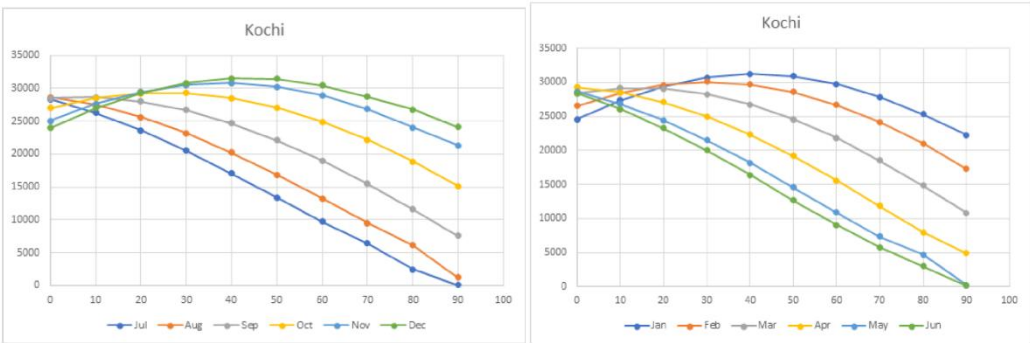


Fig 4.2: Monthly average daily total solar radiation on a tilted surface for the months of January-December (Kochi)

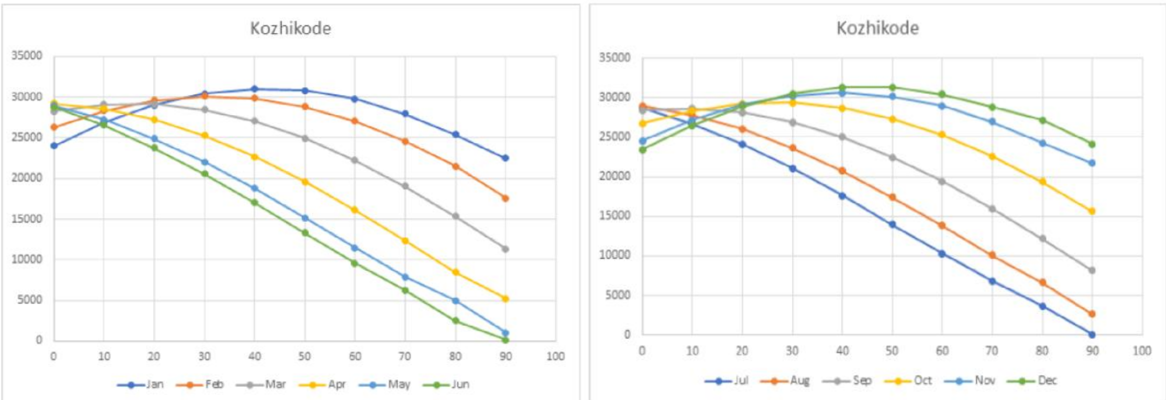


Fig 4.3: Monthly average daily total solar radiation on a tilted surface for the months of January-December (Kozhikode)

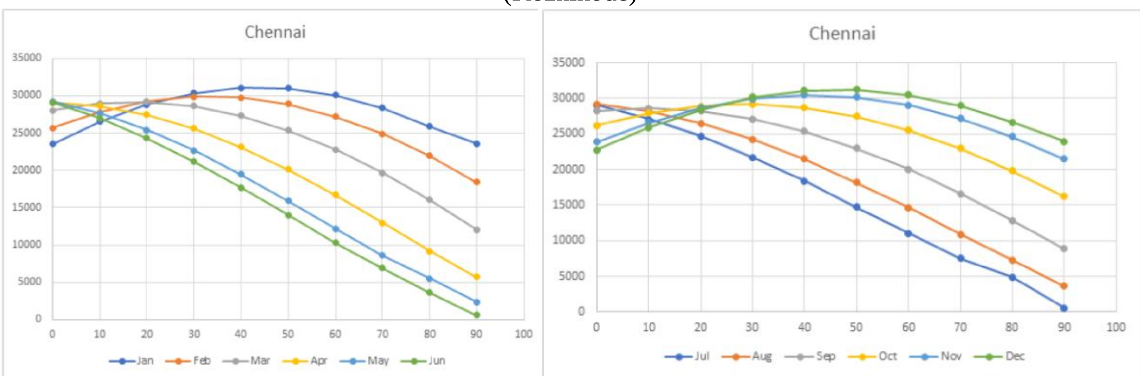


Fig 4.4: Monthly average daily total solar radiation on a tilted surface for the months of January-December (Chennai)

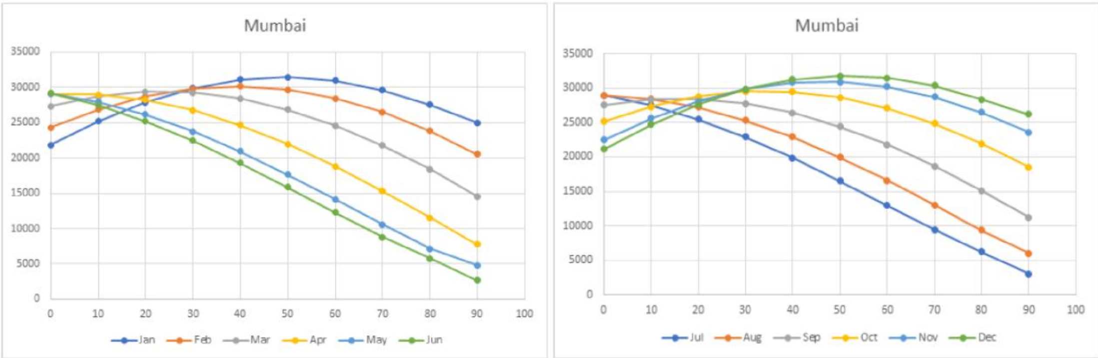


Fig 4.5: Monthly average daily total solar radiation on a tilted surface for the months of January-December (Mumbai)

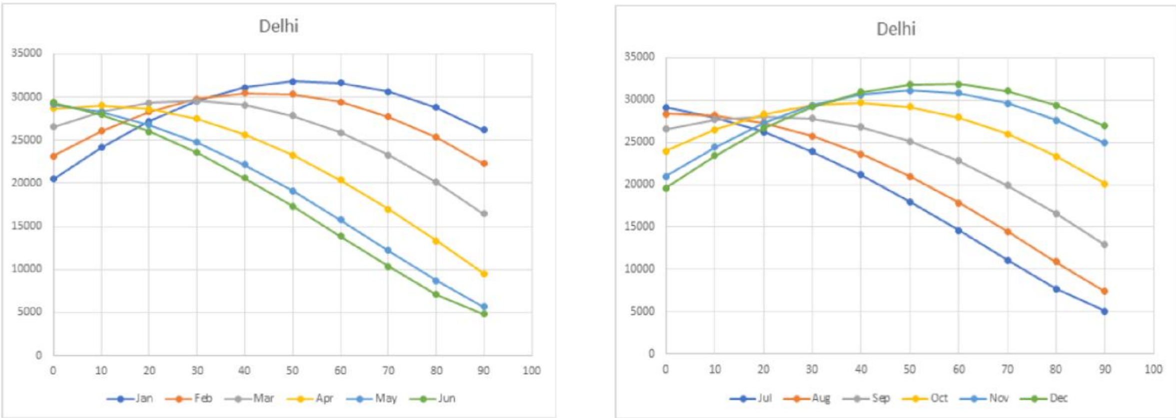


Fig 4.6: Monthly average daily total solar radiation on a tilted surface for the months of January-December (Delhi)

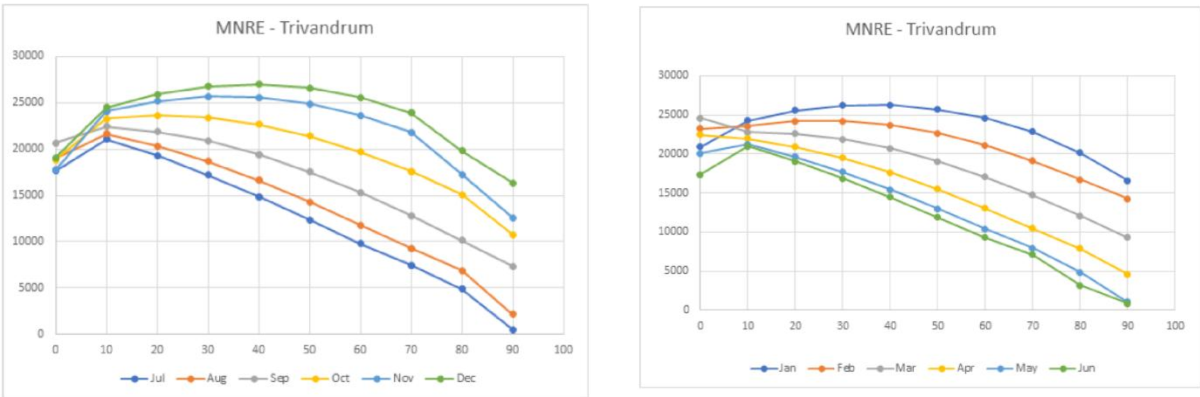


Fig 4.7: Monthly average daily total solar radiation on a tilted surface for the months of January-December in Trivandrum (MNRE)

Location	Trivandrum	Kochi	Kozhikode	Chennai	Mumbai	Delhi
Yearly Tilt	15.58	16.31	17.07	18.14	21.9	27.9
Radiation	29382.08	29776.23	300061.38	30728.12	32284.29	33780.45

Table 4.2: Yearly average radiation (kJ/m^2) and yearly average optimum tilt

Month	H_g(mnre)	H_g(calc)	H_d(mnre)	H_d(calc)	H_b(mnre)	H_b(calc)
Jan	5.8	6.97	1.47	0.92	6.99	6.05
Feb	6.46	7.51	1.54	0.99	7.45	6.51
Mar	6.83	7.93	1.72	1.05	7.35	6.88
Apr	6.24	8.06	2.08	1.06	5.89	6.99
May	5.57	7.92	2.13	1.04	4.95	6.87
Jun	4.83	7.78	2.17	1.03	3.89	6.75
Jul	4.91	7.83	2.19	1.04	3.96	6.79
Aug	5.26	7.95	2.23	1.05	4.31	6.9
Sep	5.74	7.94	2.11	1.05	5.15	6.89
Oct	5.24	7.55	2.01	0.99	4.78	6.55
Nov	4.94	7.05	1.81	0.93	4.95	6.11
Dec	5.3	6.76	1.54	0.89	6.19	5.87

Table 4.3: Comparison of MNRE data and data at Trivandrum (kW/m^2)

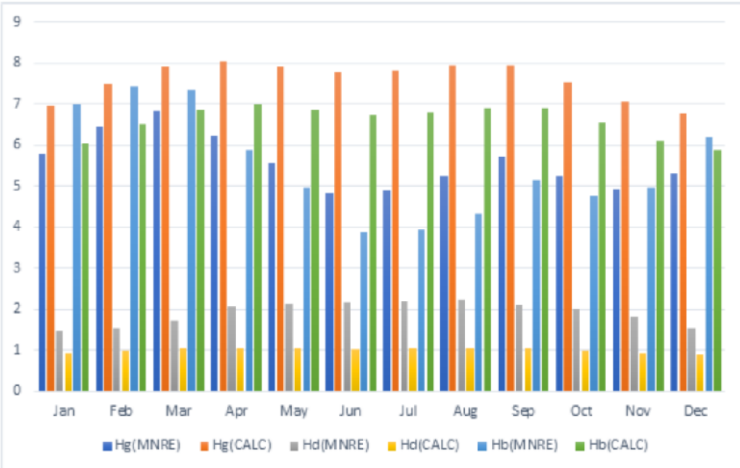


Fig 4.8: Comparison of MNRE data and Calculated at Trivandrum (kW/m^2)

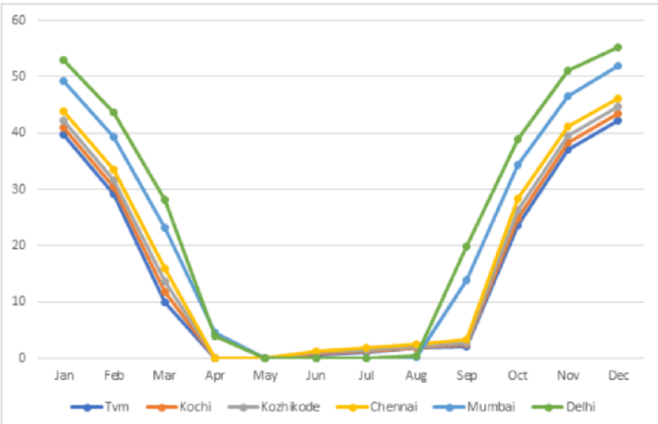


Fig 4.9: Optimum average tilt angle for each month of the year for 6 different locations

4.2 Discussion

Figure (4.1 - 4.7) shows the monthly average daily total solar radiation on a tilted surface for the months of January - December for 6 Indian locations. From these graphs, an optimum tilt for each month has been obtained where the maximum peak solar radiation is received for that particular month. Table (4.2 - 4.3) summarizes the optimum tilt angle for each month at 6 different Indian locations. In Trivandrum, the optimum tilt angle for January is 39.7, and the total monthly solar radiation falling on the surface is 30120.32 kJ/m^2. The optimum tilt in April is a minimum of 0, and the total solar radiation is 29019.50 kJ/m^2. During winter, the optimum tilt increases to a maximum of 42.2, and the radiation received is 34046.19 kJ/m^2. The yearly average optimum tilt obtained is 15.58, and the maximum radiation received is 29382.08 kJ/m^2. Similarly, in Delhi, during January, the optimum tilt obtained is 55.3, and the monthly solar radiation obtained is 42280.36 kJ/$m^{2,}$ and during the months of June, the optimum tilt comes to a minimum of 0 and radiation obtained is 29367.92 kJ/$m^{2,}$ and in winter months the tilt angle obtained is 51.2 in December which collects39405.16kJ/m^2 solar radiation monthly. The yearly average optimum tilt angle obtained is 27.9, where 33780.75 kJ/m^2 maximum radiation is received. Figure 4.9 shows the tilt angle for each month of the year for a south-faced panel at different locations in India. Table 4.15 shows the yearly average radiation and its optimum tilt for each location; it is seen that Trivandrum shows the least yearly optimum tilt and Delhi the highest, along with its average radiation. A comparison study has been shown in Table 4.16 between actual data (MNRE) and calculated data of Trivandrum. The Calculated data shows more radiation than that of actual data as less attenuation takes place in clear sky, so maximum radiation is received on the earth's surface under the conditions of clear sky.

5. Conclusion

Using the relevant equations provided above and the methods described in this study, the monthly, seasonal and yearly optimum tilt angles with the radiations received were determined. The energy produced from the solar panel can be increased with the tilt angle obtained for each month and year. The average yearly maximum radiation can be obtained with the tilt angle determined, which is approximately equal to that latitude of the location. It is observed that from the 6 locations (Trivandrum, Kochi, Kozhikode, Chennai, Mumbai and Delhi), Delhi shows the highest yearly optimum tilt and the maximum yearly radiation and Trivandrum the least. A comparison study has been done between the actual and calculated data (MNRE). It is inferred that the radiation of the calculated data received 0.75% more than that of the actual data (MNRE). With the above result, we could choose the best optimum tilt for the specific location required for domestic and industrial applications. Domestic applications such as solar water heater appliances, panels etc., in buildings. In the future, we could develop a self-adjusting motorized with solar tracking on a solar panel using microprocessor-based systems.

Reference

[1] M. Nfaoui and K. El-Hami, "Extracting the maximum energy from solar panels," *Energy Reports*, vol.4, pp. 536–545, 2018.

[2] S. P. Sukhatme and J. Nayak, *Solar energy*. McGraw-Hill Education, 2017.

[3] J. A. Duffie and W. A. Beckman, "Solar engineering of thermal processes. hoboken,"2013.

[4] P. Cooper, "The absorption of radiation in solar stills," *Solar energy*, vol.12, no.3, pp.333–346, 1969.

[5] S. Klein, "Calculation of monthly average insolation on tilted surfaces," Wisconsin Univ., Madison (USA). Solar Energy Lab., Tech. Rep., 1976.

[6] V. Modi and S. Sukhatme, "Estimation of daily total and diffuse insolation in india from weather data," *Solar Energy*, vol. 22, no.5, pp.407–411, 1979.

[7] H. Garg and S. Garg, "Correlation of monthly-average daily global, diffuse and beam radiation with bright sunshine hours," *Energy Conversion and Management*, vol. 25, no.4, pp. 409–417, 1985.

[8] B. Y. Liu and R. C. Jordan, "The interrelationship and characteristic distribution of direct, diffuse and total solar radiation," *Solar energy*, vol.4, no.3, pp. 1–19, 1960.

[9] M. Jamil Ahmad and G. N Tiwari, "Optimization of tilt angle for solar collector to receive maximum radiation," *The open renewable energy journal*, vol.2, no.1, 2009.