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Natural Convection in Rectangular Enclosure Heated from Below and Cooled from the Side

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Abstract: Steady natural convection of air in a two-dimensional enclosure, isothermally heated from below and cooled from the side, is studied numerically. Based on numerical predictions, the effects of Rayleigh number and aspect ratio on flow pattern and energy transport are investigated for Rayleigh numbers ranging from 10^3 to 10^6 and for 3 different aspect ratios 1, 3, 5.Numerical solutions are obtained for the Navier Stokes equations and Energy equation on a staggered grid using a finite volume scheme. The boundary conditions are uniform wall temperature and insulation. Numerical results are obtained for various values of Rayleigh number and are presented in the form of streamlines, isotherm contours and local Nusselt number as a function of Rayleigh number. The effect of the Rayleigh number on heat transfer is found to be more significant when the enclosure is with aspect Ratio > 1. A correlation connecting the Nusselt number, Rayleigh number and Aspect ratio is developed for convective heat transfer based on numerical data. The effect of the increase in aspect ratio is analysed.

Keywords: Natural convection, Finite volume method, Aspect ratio, Rayleigh Number.

I. INTRODUCTION

The phenomenon of natural convection in fluid-filled rectangular enclosures has received considerable attention in recent years. This is mainly because it affects thermal performance in many engineering applications, such as building thermal design, nuclear reactor design, solar energy collectors, etc. Therefore, natural convection is preferred in several fields where the heat to be dissipated is low enough; itis an attractive system in thermal control because of its low cost, reliability and simplicity in use.

Natural convection in closed cavities has been studied primarily in fluid layers heated from below or fluid layers heated from the side. Among the earlier studies, Ganzarolli and Milanez [1] studied natural convection in a rectangular enclosure heated from below and cooled along a single or both sides. November and Natsteel [4] have shown a specific interest in focusing natural convection within a rectangular enclosure wherein bottom heating and/or top cooling are involved. They observed that the heated layer adjacent to the lower surface remains attached up to the turning corner though the density stratification in the layer is unstable. Recently B. Calcagni et al [5] studied natural convection in a square enclosure heated from below experimentally and numerically. Further Orhan Aydin et al. [2] have also reported natural convection in rectangular enclosures heated from one side and cooled from the ceiling.N. Ramesh et al. [7] studied experimentally and numerically the effect of boundary conditions on natural convection in an enclosure with differentially heated isothermal vertical walls and horizontal bounding walls. M. Sathiyamoorthy et al. [9] numerically studied steady natural convection flows in a square cavity with linearly heated side walls. O. Polatet al. [10] studied conjugate heat transfer in inclined open shallow cavities. They numerically analyzed the effect of Rayleigh number and aspect ratio on the flow field and heat transfer.

This work analyses a rectangular enclosure heated from below and cooled from one side. The boundary condition for the bottom side is uniform temperature, while the top and the other vertical sides area diabatic. The primary governing equations are solved numerically for rectangular enclosures.
With aspect ratio (L/H) ranging from 1 to 5. The finite Volume Method is used here to solve the differential equations. The flow field is solved by the SIMPLE (Semi Implicit Method for Pressure Linked Equations) algorithm. A correlation for the Nusselt number is developed in this study.

II. PROBLEM DESCRIPTION

A schematic of the two-dimensional problem showing geometrical and boundary conditions is given in fig. 1.Constant temperature, T_h is applied on the bottom surface, and the cavity is cooled from the vertical side wall (left) kept at temperature T_c.

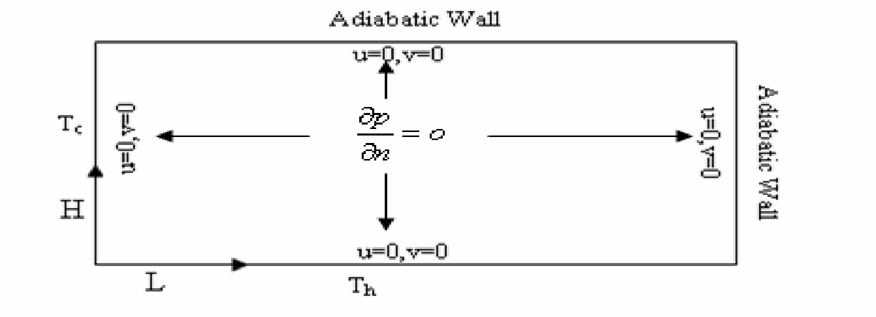


Fig. 1 Schematic of the problem geometry with boundary conditions

III. MATHEMATICAL FORMULATION

Steady state condition is considered here, and hence the properties of the fluid in the cavity are assumed to be constant. The effect of density variations is accounted using Boussinesq approximation, wherein an extra term is added as body force in the negative y- direction of v- momentum equation. This way, the temperature field is coupled with the flow field. The governing equations for the steady natural convection flow using conservation of mass, momentum, and energy can be written as,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta\left(T-T_c\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

With boundary conditions

$$u\left(x,0\right)=u\left(x,L\right)=u\left(0,y\right)=u\left(L,y\right)=0$$
$$v\left(x,0\right)=v\left(x,L\right)=v\left(0,y\right)=v\left(L,y\right)=0$$
$$T\left(x,0\right)=T_h,\frac{\partial T}{\partial x}\left(x,L\right)=0$$
$$T\left(0,y\right)=T\left(0,L\right)=T_c \tag{5}$$

Here x and y are the distances measured along the horizontal and vertical directions respectively; u and v are the velocity components in the x- and y- directions respectively; T denotes the temperature; p is the pressure and ρ is the density; T_h and T_c are the temperature at hot and cold walls respectively; L is the side of the cavity. Using the following non-dimensionalisation:

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{uL}{\alpha},V=\frac{vL}{\alpha},\theta=\frac{T-T_c}{T_h-T_c},$$
$$P=\frac{pL^2}{\rho\alpha^2},\Pr=\frac{\nu}{\alpha},Ra=\frac{g\beta\left(T_h-T_c\right)L^3\Pr}{\nu^2} \tag{6}$$

The governing equations (1)-(4) reduce to non-dimensional form as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{8}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta \tag{9}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{10}$$

With the boundary conditions

$$\begin{aligned}
&U(X,0)=U(X,1)=U(0,Y)=U(1,Y)=0,\\
&V(X,0)=V(X,1)=V(0,Y)=V(1,Y)=0,\\
&\theta(0,1)=0,\frac{\partial\theta}{\partial X}(1,Y)=\frac{\partial\theta}{\partial Y}(X,1)=0,\\
&\theta(X,0)=1,\theta(0,1)=0
\end{aligned} \tag{11}$$

Here X and Y are dimensionless coordinates varying along horizontal and vertical directions respectively; U and V are dimensionless velocity components in the X- and Y-directions respectively; θ is the dimensionless temperature; P is dimensionless pressure; Ra and Pr are Rayleigh and Prandtl numbers respectively.

The local Nusselt numbers for heated side wall and cold side wall respectively are given by

$$\mathrm{Nu_l}(\mathrm{Y})=\left(\frac{\partial\theta}{\partial X}\right)_{X=0}\frac{1}{(T_h-T_c)} \tag{12}$$

$$\mathrm{Nu_r}(\mathrm{Y})=\left(\frac{\partial\theta}{\partial X}\right)_{X=1}\frac{1}{(T_h-T_c)} \tag{13}$$

IV. NUMERICAL PROCEDURE

The governing equations along with the boundary conditions are solved numerically using finite volume method. The transport equations are discretized over a staggered grid using upwind scheme for the convection terms, central difference scheme for diffusion terms. The velocity pressure linkage is solved using the SIMPLE (Semi Implicit Method for Pressure Linked Equations) algorithm [17].The solution of the discretized momentum and pressure correction equation is obtained by the line by line TDMA (Tri Diagonal Matrix Algorithm) [17].Under relaxation parameters are introduced for velocities and temperature to control the advancement of the solution field until a steady final state was obtained. Starting from arbitrarily specified initial values of variables, the discretized equations are solved until an asymptotic steady state solution is reached. The convergence criterion is that the maximal residual of all the governing equation is less than 10^{-6}.

A. Grid Independence Study

A non uniform mesh with a large concentration of nodes in regions of steep gradients, such as close to the solid portion is employed. A grid independence study is conducted using four different grid sizes of 9×9, 41× 41, 61× 61 and 81× 81(uniform and no uniform) for validation problem and it is observed that a further refinement of grids from 41×41 to 61× 61 does not have a significant effect on the results in terms of the maximum value of stream function and Nussselt number. Grid independence study is conducted and is tabulated in Table 1.Based on this observation, a uniform grid of 41×41 is used for all the calculations for validation problem. For rectangular enclosures with aspect ratio 1, 3 and 5, the grid sizes using are 81×81, and 151×51and151×31 respectively.

Table 1 Grid-independence study results for Ra=10^5

Aspect Ratio	(NX,NY)grid	Ψ_{max}	Nu
Validation problem	41×41 61×61 81×81	10.301 10.091 10.025	4.559 4.598 4.589
AR=1	41×41 61×61 81×81	18.2474 18.3827 18.4414	6.723 6.969 7.1
AR=3	91×31 121×41 151×51	17.0363 16.9258 16.8533	8.072 8.218 8.241
AR=5	1261×26 151×31 201×41	17.0660 16.9051 16.7894	8.242 8.335 8.347

V. VALIDATION

The present code is validated for natural convection heat transfer by comparing the results of buoyancy driven laminar heat transfer in a square cavity with differentially heated vertical walls. The left wall was kept hot while the right wall was cooled. The top and bottom walls are insulated. In the present work numerical predictions, using the early mentioned algorithm, have been obtained for Rayleigh number between 10^3 and 10^6 on anon-uniform 41× 41 grid points. A very good agreement is obtained between the present problem solution and the bench mark solution of de Vahl Davis [11]. The results of comparison are presented in Table 2a and 2b.It is seen that the deviation vary from 5-6 % for ψ_{max} and 0.9 – 9 % for Nusselt number.

Table 2aComparison of Numericalresults (41×41 grid)

Ra. No	Present Study			de Vahl Davis[11]		
	Ψ_{max}	Ψ_{mid}	Nu_l	Ψ_{max}	Ψ_{mid}	Nu
10^3	1.160	1.159	1.124	-	1.174	1.116
10^4	5.088	5.08	2.559	-	5.098	2.234
10^5	10.082	9.752	4.559	9.739	9.234	4.487
10^6	18.160	17.857	9.273	17.613	17.15	8.811

Table 2bComparison of Numerical results

% Deviation		
Ψ_{max}	Ψ_{mid}	Nu
-	1.303	0.686
-	0.353	1.164
3.522	5.605	1.605
5.889	4.124	5.247

VI. RESULTS AND DISSCUSSION

A. Square enclosure-Validation

Fig. 2and 3 illustrates the stream function and isotherms for various values of Ra = 10^3 – 10^6 and Pr = 0.7 with uniformly heated left vertical wall and cooled right vertical wall where top and bottom walls insulated. Due to uniformly heated side wall fluid heated, rises up go to right side and flow down along the wall forming a clockwise roll inside the enclosure. As the value of Rayleigh number increases and consequently the circulation inside the cavity, the warmer fluid tends to occupy the upper side, compressing the isotherms near the vertical walls. For increasing values of the Rayleigh number the temperature tends to be more uniform in the upper region of the enclosure.

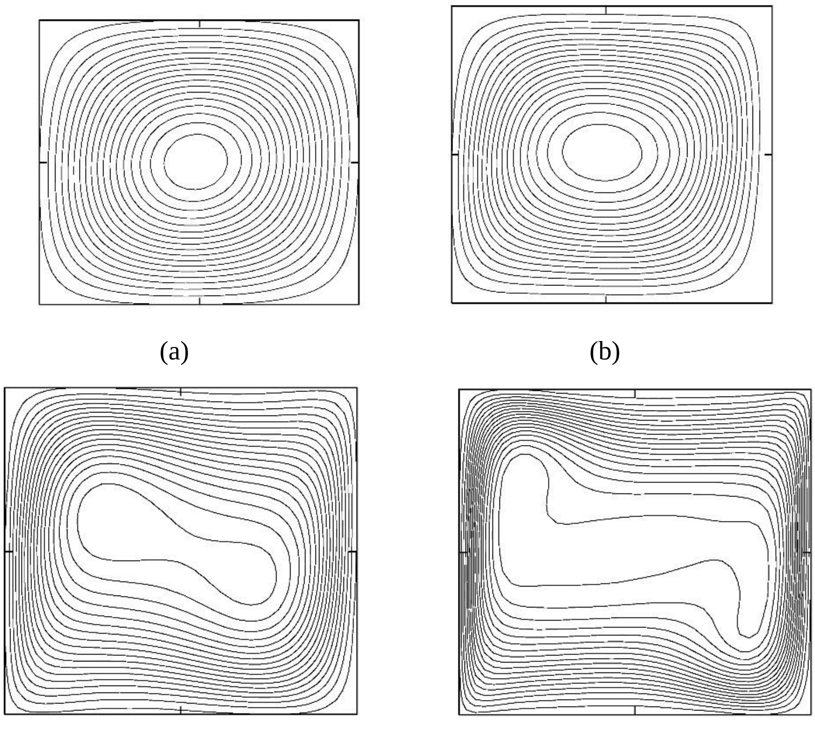
B. Square enclosure

Fig. 4 and 5 illustrates the stream function and isotherms for values of Ra=10^3-10^6 for the square enclosure (AR=1) with uniformly heated bottom wall, cooled left vertical wall and top and right insulated walls. At Ra=10^3, recirculation fills the entire cavity with maximum value of stream function equal to 0. 9501. When Rais increased strength of recirculation increases. Maximum values of stream functions are 6.235, 18.441, 32.999 for Ra=10^4,10^5and10^6respectively. Beyond Ra=10^4 ,the fluid rising over the vertical walls after getting heated by horizontal wall separates near the right top corner of the cavity. The fluid flows leftwards over the top wall gets displaced due to the effect of boundary layer at the topwall. By the effect of cooling of left vertical wall, the streamlines approaching the cold wall at the left upper corner. When Ra=10^5 isotherms fills the entire cavity indicating that the major mode of heat transfer is conduction. As Ra increases, the thermal boundary layer over the left and bottom walls becomes thinner.

Fig.6 and 7 illustrates the stream function and isotherms for various values of Ra = 10^3- 10^6 for the rectangular enclosure (AR=3).At Ra = 10^3,recirculation fills the entire cavity with maximum value of stream function equal to 1.306.When Rais increased strength of recirculation increases. Maximum values of stream functions are 6.612, 16.853, 44.003 for Ra = 10^4, 10^5 and 10^6 respectively. Beyond Ra =10^4,the fluid rising over the vertical walls after getting heated by horizontal wall separates near the right top corner of the cavity. The fluid flows leftwards over the top wall gets displaced due to the effect of boundary layer at the topwall. By the effect of cooling of left vertical wall, the streamlines approaching the cold wall at the top region of cavity tends to more upwards in the left upper corner. When Ra = 10^3 isotherms fills the entire cavity indicating that the major mode of heat transfer is conduction. As Ra increases, the thermal boundary layer over the left and bottom walls becomes thinner.

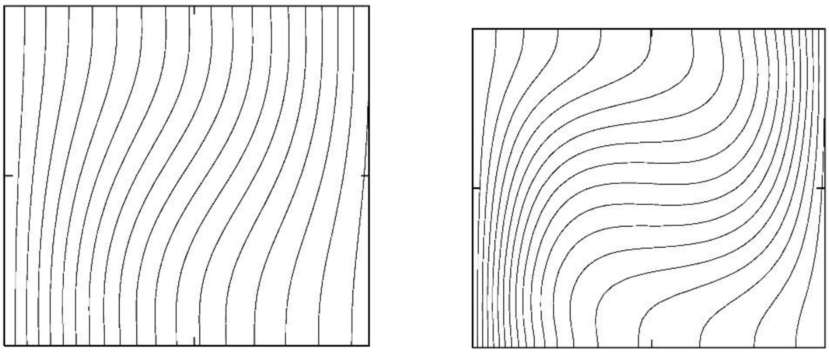
Fig.8 and 9 illustrates the stream function and isotherms for various values of Ra = 10^3 -$10^6$for the square enclosure(AR=5).At Ra = 10^3,recirculation fills the en- tire cavity with maximum value of stream function equal to 1.295.When Rais increased, the strength of recirculation increases. Maximum values of stream functions are 6.578, 16.906,and 44.545 for Ra = 10^4, 10^5 and 10^6, respectively. Beyond Ra = 10^4,the fluid rising over the vertical walls after getting heated by horizontal wall separates near the right top corner of the cavity. The fluid flows leftwards over the top wall gets displaced due to the effect of the boundary layer at the topwall. By the effect of cooling of the left vertical wall, the streamlines approaching the cold wall at the top region of the cavity tend to move upwards in the left upper corner. When Ra = 10^3, isotherms fill the entire cavity indicating that the major mode of heat transfer is conduction. As Ra increases, the thermal boundary layer over the left and bottom walls becomes thinner.

Fig.10 shows the relation connecting Rayleigh No and Nusselt No.



(a) (b) (c) (d)

Fig. 2 Streamlines for Pr=0.7. (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6



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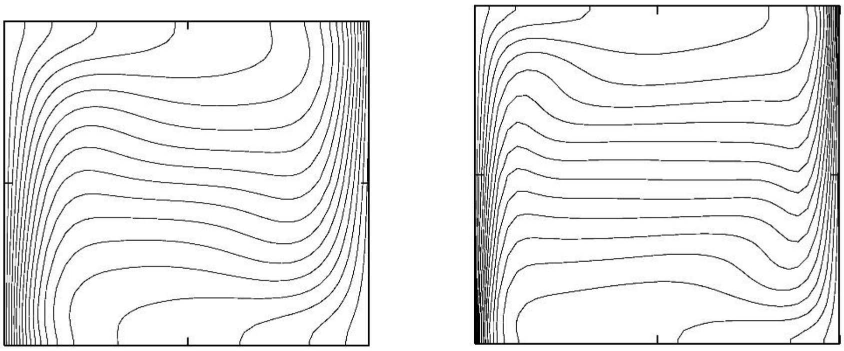
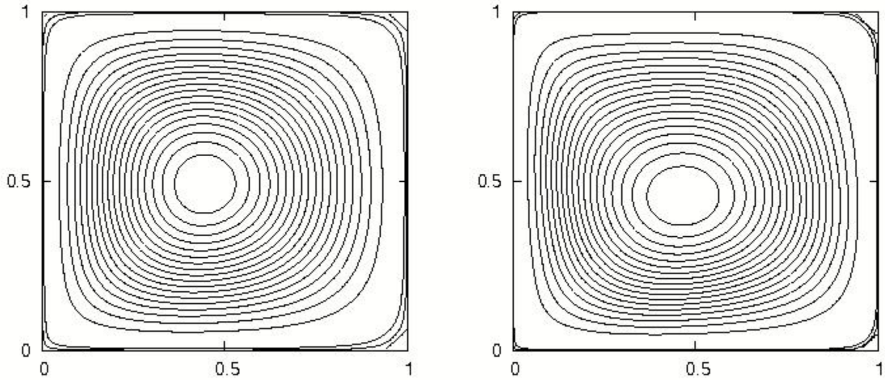
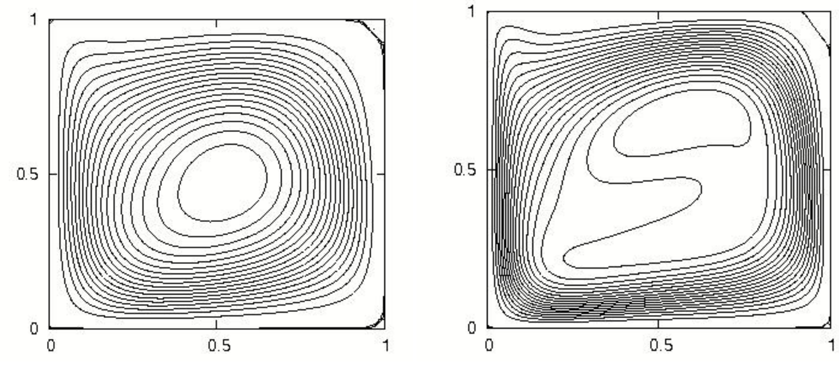


(c) (d)

Fig. 3. Isotherms for Pr=0.7.(a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6

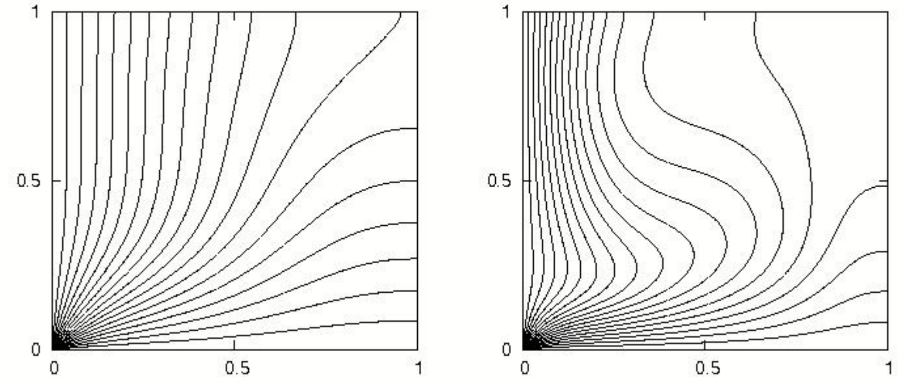


(a) (b)

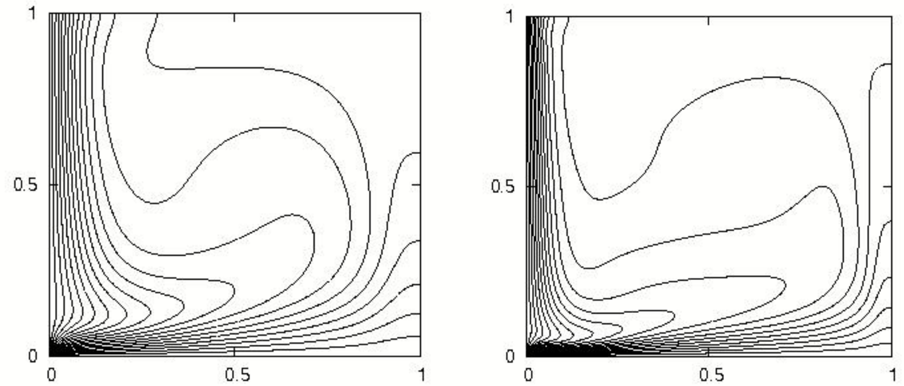


(c)(d)

Fig. 4. Streamlines for AR=1 (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6

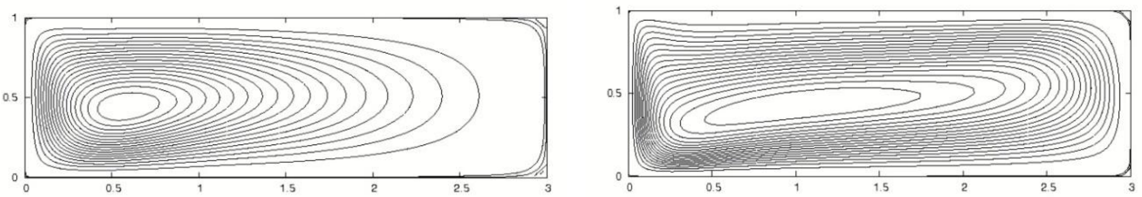


(a) (b)



(c)(d)

Fig. 5. Isotherms for AR=1 (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6



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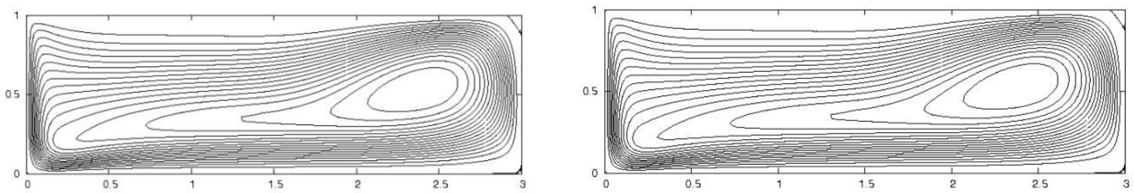
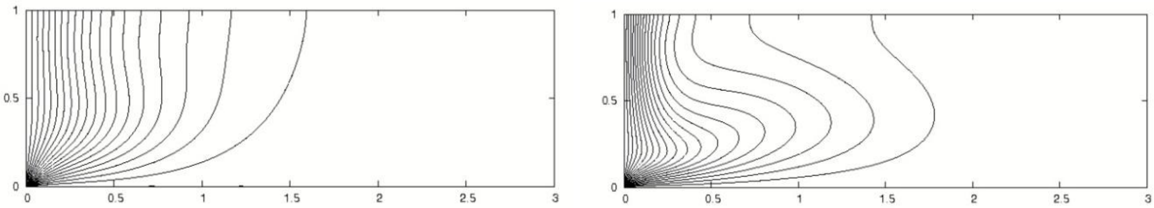


(c) (d)

Fig. 6. Streamlines for AR=3 (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6



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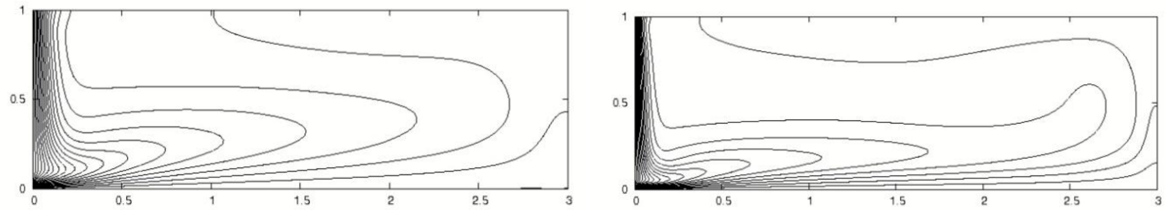
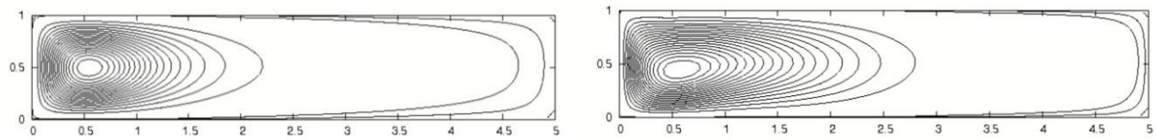
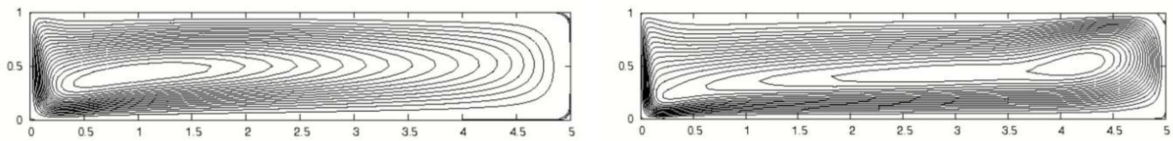


(c) (d)

Fig. 7. Isotherms for AR=3 (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6

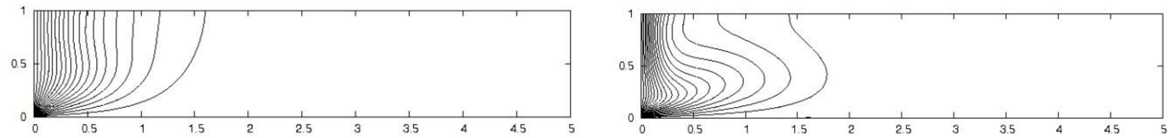


(a) (b)

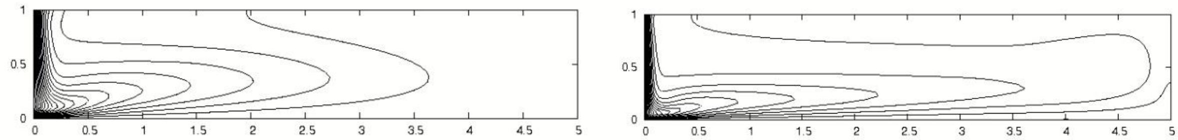


(c) (d)

Fig. 8 Streamlines for AR=5 (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6



(a) (b)



(c) (d)

Fig. 9 Isotherms for AR=5 (a) Ra=10^3 (b) Ra=10^4(c) Ra=10^5 (d) Ra=10^6

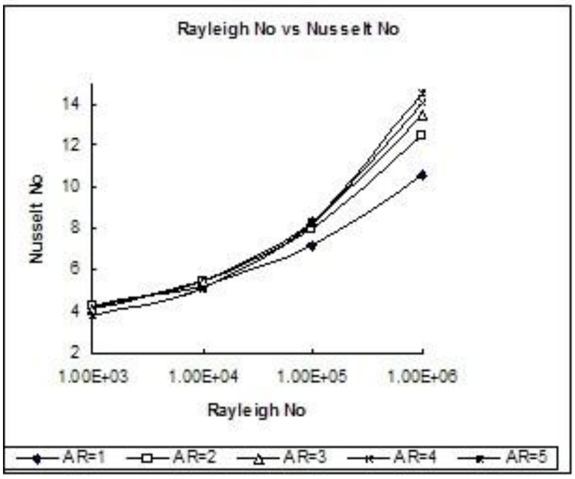


Fig.10 Rayleigh No Vs Nusselt No

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C. Correlation

The correlation developed for aspect ratio 1 to 5 is

$$Nu = 1.13\, AR^{0.05} Ra^{0.17} \qquad (14)$$

The correlation indicates the least influence of increase of aspect ratio on Nusselt number

VII. CONCLUSION

A numerical study of natural convection in a rectangular enclosure heated from bottom and cooled from left side wall with adiabatic top and right walls is conducted. The computer code is developed in FORTRAN based on Finite Volume Method. The code is validated against one bench mark problem for a square enclosure. Numerical study on rectangular enclosure with aspect ratio 1, 3 and 5 is conducted. The results are shown as streamlines and isotherms. The correlation developed indicates the least influence of increase of aspect ratio on Nusselt number. The increase of Aspect ratio has got least influence in convective heat transfer.

VIII. NOMENCLATURE

k thermal conductivity, W m^{-1} K^{-1}
L side of the square cavity, m
N total number of nodes
Nu local Nusselt number
Nu_l local Nusselt number at the left wall
Nu_r local Nusselt number at the right wall
Nu average Nusselt number
p pressure, Pa
P dimensionless fluid pressure
Pr Prandtl number

Ra Rayleigh number
T fluid temperature, K
T_h temperature of hot wall, K
T_c temperature of cold wall, K
u x component of velocity
U x component of dimensionless velocity
v y component of velocity
Y y component of dimensionless velocity
X dimensionless distance along x-coordinate
Y dimensionless distance along y-coordinate
a_r/AR aspect ratio(Length of the geometry/Height of the geometry)

Greek symbols
α thermal diffusivity, m^2 s^{-1}
β volume expansion coefficient, K^{-1}
θ dimensionless temperature
υ kinematic viscosity m^2 s^{-1}
ρ density, kg m^{-3}
Ψ stream function

Subscripts
b bottom wall
c cold wall
h hot wall
r right wall
l left wall
s side wall

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