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Analysis of the System Stability for an anti-Sway method relative to Harbour Cranes

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Abstract: The requests relatively to the large-sized mobile harbor cranes are increasing. Typically, harbour cranes are self-traveling but do not require any additional civil works to improve the bearing strength of foundation soil. In this work, we study the stability of a model for controlling the Sway of a Harbour crane. The multiple-input and multiple-output (MIMO) mechanical system is modeled by a set of nonlinear differential equations in which the mathematical model is divided into two subsystems: the first one for actuated outputs and the second one for underactuated outputs. The nonlinear feedback of the states is used to linearize the closed-loop system. The control system is defined by linearly combining two different parts that are separately obtained from the actuated and unactuated system. Hurwitz's criterion was applied to investigate the system stability for the equations describing the harbour crane dynamics. We reach the result to obtain the stability criteria for the "zero" dynamics relative to the harbour crane model.
Keywords: Harbour crane, Underactuated mechanical systems, Stability analysis, Nonlinear feedback control.

1 Introduction

Cranes are very important machinery that has been strongly used for container handling in the harbour and object shifting on construction sites. Generally, cranes can be subdivided into two most important groups: gantry cranes and boom cranes. Boom cranes are industrial structures that are used in building construction, factories, harbours, and shipyard. Besides, they are largely used to transport heavy loads in shipyards, factories, and high building construction. These cranes are usually controlled manually where operators use a joystick and an accelerator pedal to control the movements and direction of the cranes.

Over the years, a lot of studies have been focused on the development of efficient controllers for gantry cranes. However, only a limited number of studies have been carried out to design control approaches to reduce the payload sway of boom cranes.

The recent work of Ramli et al. [1] allows having an exhaustive literature review of the strategies relative to the crane control and the relative published works. For example, concerning the studies relating to Anti-sway for Overhead and gantry cranes, we can mention, among many others, the works [2] and [3].

Feedback and feed-forward control strategies are two major sway control schemes that can be utilized. Feedback control techniques use measurement and estimations of the system states to attenuate the swaying of the system. Feedback controllers can be designed to be robust to parameter uncertainties.

On the other hand, feed-forward techniques for sway attenuation involve developing the control input through consideration of the physical and swaying properties of the system, so that system swaying at dominant response modes is reduced. This method does not require additional sensors or actuators and does not account for changes in the system once the input is developed. For boom cranes, feed-forward and feedback control techniques can be used for sway attenuation and position control respectively. Only a few works have considered anti-swing for boom (slewing) cranes. This is due to the complexity of the calculations for the motion control.

The recent works of the author [4] and [5] are examples of feed-forward techniques for sway attenuation for boom (tower) cranes.

With the recent trend toward containerization, it is more and more important for local small and medium-sized harbors to use mobile harbor cranes that are capable of handling containers as well as other general cargoes in different ways. Therefore, demands for large-sized mobile harbor cranes, which are self-traveling and versatile but do not require any additional civil works to increase the foundation soil, are increasing. We are developing a method for harbor crane systems which has an anti-sway function.

In the last years, some works have been done in order to control the sway of the payload for a Harbour (boom) crane.

Mainly, the control schemes can be divided into open and closed-loop control approaches. Feedback control focuses on disturbance rejection and enables precise automatic positioning, while open-loop systems have primarily the purpose of reducing unwanted payload oscillations for a given reference.

Some works concern the study of harbor cranes in the open sea. We can see, in example, the recent work of D. Kim et al. [6]. This kind of work is extremely complex because it is necessary to take into account also disturbances such as waves and wind. In this case, the Mobile Harbor has 6 degrees of freedom motion caused by external disturbances such as waves and wind.

We will limit ourselves to the study of Harbor cranes on the fixed surface, on land, not on the sea. Simplifying the problem in this case.

In particular, we want to point out four recent and important works regarding Harbor cranes on the land. The works of E. Arnold et al. [7], J. Neupert et al. [8], T. Toyohara et al. [9] and D. Kim et al. [10] address the problem of studying an optimal control approach or an alternative feedback control system.

As part of the study relative to the anti-sway control of harbor crane systems, we investigated the stability of the solution we defined. In fact, only when the solution defined is stable will we be able, in a subsequent work, to discuss its performance and operating characteristics.

In general, our solution for controlling a harbor crane is in the context of underactuated systems. In practice, many control problems involve the â€œunderactuatedâ€ behavior of mechanical systems. In underactuated systems, the number of equipped actuators is less than that of the controlled variables. That is, actuators do not directly control some degrees of freedom. Within the scope of Harbour crane system we studied, the underactuated variables turn out to be the "sway" variables, that is the sway angles in tangential and normal direction to the slewing rotation of the boom.

We will refer, in the analysis of the stability of the anti-sway system related to Harbor cranes, to the following fundamental works. The works we cite and use are those of L.A. Tuan et al. [11], L.A. Tuan et al. [12] and M.W. Spong [13].

E. Lefeber et al. [14] investigated tracking control for underactuated ships in which three state variables, that is surge, sway, and yaw, were controlled by only two inputs: surge force and yaw torque.

This paper is organized as follows.

In Section 2 the dynamical model of the Harbour crane is described. The harbour crane is modeled defining a multi-body system including base, mast (fixed vertical column), jib, trolley and payload. From Lagrange equations, the five dynamical equations relative to the two sway angles (underactuated system) and to theother lagrangian variables (actuated system) are obtained, including also the dissipation function. By these equations, the designed control law for the presented model is obtained.

In Section 3, a mathematical description of an Underactuated system and Stability Analysis for the some system are given.

In Section 4 an application of the theory defined in Section 3 is given, developping the matrix equation for an Harbour crane system and arriving to analyze the stability of the control equations. In this way, we obtain the goal to establish the constraint conditions which result necessary to define the stability of the system.

At the end, in Section 5 concluding remarks are defined.

2 Modelling of an Harbour crane

An Harbour (boom) crane is a multi-body system including base, mast (fixed vertical column), jib, trolley, and payload. The boom crane system considered in this work is shown in Figure 1 and it is schematized in Figure 2, where x and z represent the base coordinates. The crane system consists of a fixed vertical column, a rigid boom link, a hoisting line, and a payload. φ, θ, and l represent the slew angle, luff angle, the length of the hoisting line respectively. The slew angle is the rotary angle of the hub of the boom crane or slewing pedestal controlled by the operator’s slew command whereas the luff angle is the elevation or luffing angle of the boom link. Sway angles are excited as the system operates, namely the tangential sway φ_t, and the radial sway φ_r. In this study, the payload is regarded as a point mass and the system exhibits the behavior of a pendulum. In the described system with at least one first and one second strand of cables, both strands of cables extend from the tip of the boom to a suspension element such as a hook. The length of the cable can be adjusted by a corresponding drive, in order to move the load in the vertical direction.

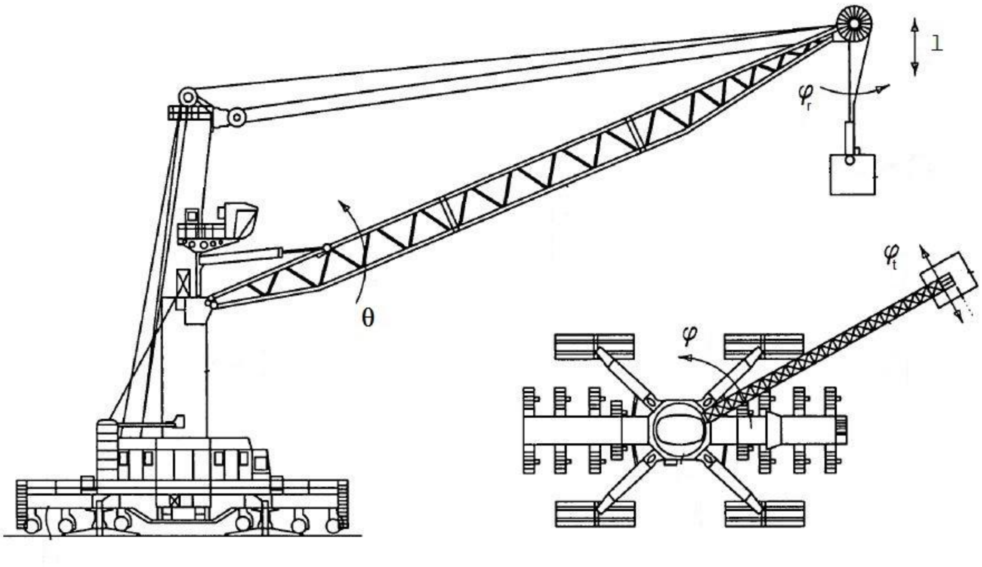

Figure 1: General view of an Harbour crane. Side and top view.

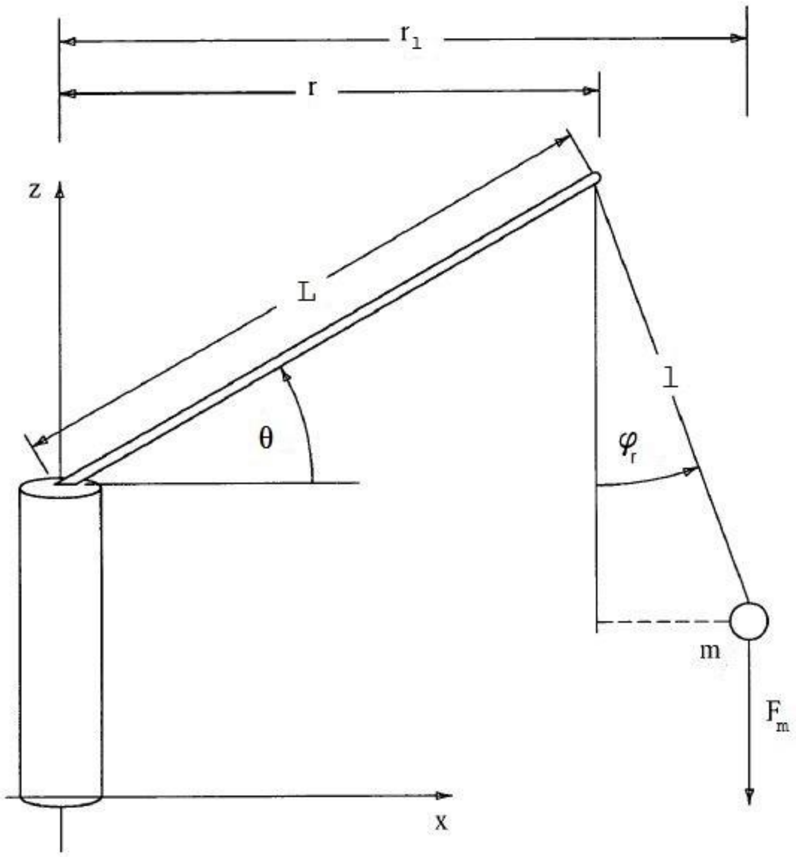

Figure 2: Geometric description of the harbour crane system.

With reference to Fig.2, the dynamics of the generalized Harbour crane is represented as a multi-body system with five independent degrees of freedom, described by five Lagrangian coordinates q_i :

$q_1 = r$: radial position of the boom on the axis x;

$q_2 = \varphi$: slewing angle (rotation around the z axis);

$q_3 = l$: hoisting length of the cable (along the z axis);

$q_4 = \varphi_r$: sway angle in the radial direction;

$q_5 = \varphi_t$: sway angle tangential to the trajectory of the slewing direction;

3 Mathematical description of an Underactuated system and Stability Analysis

Basically, we refer to the paper of L.A. Tuan et al. [11]. In general, the physical behavior of a MIMO mechanical system is governed by a set of differential equations of motion. Consider an underactuated system with n degrees of freedom driven by m actuators ($m < n$). We will use the convention that matrix equations will be represented in bold.

3.1 Mathematical description of an Underactuated system

The mathematical model, which is composed of n ordinary differential equations, is simplified in matrix form as follows

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Q \tag{1}$$

where $q = [q_1, q_2, \ldots q_n]^T \in R^n$ is the vector of the generalized coordinates, and $Q \in R^n$ denotes the vector of the control inputs. $M(q) = M(q)^T = [m_{i,j}]_{n\,x\,n} \in R^{n\,x\,n}$ is the symmetric mass matrix, $C(q,\dot{q})$ =$[c_{i,j}]_{m\,x\,n} \in R^{m\,x\,n}$ is the Coriolis and centrifugal matrix, $G(q) = [g_1, g_2, \ldots g_n]^T \in R^n$ is the gravity vector. Given that the system has more control signals than actuators, Q has only m nonzero components as $U = [u_1, u_2, \ldots u_m]^T \in R^m$ being a vector of nonzero input forces. As an underactuated system, its n output signals are driven by m actuators. Its mathematical model is divided into two auxiliary dynamics, namely, actuated and unactuated systems. Correspondingly, we define the generalized coordinates q_a = $[q_1, q_2, \ldots q_m]^T \in R^m$ for actuated states and $q_u = [q_{m+1}, q_{m+2}, \ldots q_n]^T \in R^{n-m}$ for unactuated states. The matrix differential equation (1) can be divided into two equations as follows:

$$M_{11}(q)\ddot{q}_a + M_{12}(q)\ddot{q}_u + C_{11}(q,\dot{q})\dot{q}_a + C_{12}(q,\dot{q})\dot{q}_u + G_1(q) = U \tag{2}$$

$$M_{21}(q)\ddot{q}_a + M_{22}(q)\ddot{q}_u + C_{21}(q,\dot{q})\dot{q}_a + C_{22}(q,\dot{q})\dot{q}_u + G_2(q) = 0 \tag{3}$$

Matrices $M(q)$, $C(q,\dot{q})$ and $G(q)$ have the following form:

$$M(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \\ & \end{bmatrix}; \quad C(q,\dot{q}) = \begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q,\dot{q}) \\ & \end{bmatrix}; \quad G(q) = \begin{bmatrix} G_1(q) \\ G_2(q) \end{bmatrix} \tag{4}$$

Obviously, $M(q)$ is symmetric positive definite. The actuated equation 2 shows direct relationship between the actuated states q_a and the actuators U . By contrast, the unactuated equation 3 does not display the constraint between the unactuated states q_u and the inputs U . Physically, input signals U drive the actuated states q_a directly and the unactuated states q_u indirectly.

The dynamics of the system, which is characterized by equations 2 and 3, allows to define a simpler model with an equivalent linear form based on the nonlinear feedback method [13], being $M_{22}(q)$ a positive definite matrix. The unactuated states q_u can be determined from Equation 3 in the following way:

$$\ddot{q}_u = -M_{22}^{-1}(q)\{M_{21}(q)\ddot{q}_a + C_{21}(q,\dot{q})\dot{q}_a + C_{22}(q,\dot{q})\dot{q}_u + G_2(q)\} \tag{5}$$

In underactuated mechanical systems, we can take advantage of a fact: the unactuated state q_u has a geometric relationship with the actuated state q_a. Therefore, control input U indirectly acts on q_u through q_a. Considering the actuated states q_a as the system outputs, the actuated equation 2 can be â€œlinearizedâ€ by defining

$$\ddot{q}_a = V_a \tag{6}$$

with $V_a \in R^m$ as the equivalent control inputs. Control inputs U are designed to drive the actuated states q_a to the target values q_{ta}. In order to define the profiles of the state trajectories, the following equivalent control inputs are selected:

$$V_a = \ddot{q}_{ta} - K_{da}(\dot{q}_a - \dot{q}_{ta}) - K_{pa}(q_a - q_{ta}) \tag{7}$$

Given that $q_{ta} = const$, Equation 7 can be reduced into

$$V_a = -K_{da}\dot{q}_a - K_{pa}(q_a - q_{ta}) \tag{8}$$

with K_{da} and K_{pa} positive diagonal matrices in $\in R^{m\,x\,m}$. Considering Equation 7 and the definition eq. 6, the differential equation of the control error is obtained by

$$\delta\ddot{q}_a + K_{da}\delta\dot{q}_a + K_{pa}\delta q_a = 0 \tag{9}$$

where $\delta q_a = q_a - q_{ta}$ is the control error vector of the actuated states.

3.2 Stability Analysis of an Underactuated system

The dynamics of the control error (eq. 9) is exponentially stable for every $K_{da} > 0$ and $K_{pa} > 0$. In other words, the control errors of the actuated states δq_a tend to zero as the time t becomes infinite.

To obtain the stabilization of the unactuated states q_u, the nonlinear feedback method can be applied to subdynamics relative to equation 2 in the following way:

$$\ddot{q}_u = V_u = -K_{du}\dot{q}_u - K_{pu}q_a \tag{10}$$

with K_{du} and K_{pu} positive diagonal matrices in $\in R^{(n-m)\,x\,(n-m)}$. $V_u \in R^{n-m}$ are the equivalent control inputs of the unactuated states. The control input U ,defined by Equations 2 and 10 allows to ensure the stability of the unactuated states q_u because the control error dynamics, that is,

$$\ddot{q}_u + K_{du}\dot{q}_u + K_{pu}q_u = 0 \tag{11}$$

is stable for every $K_{du} > 0$ and $K_{pu} > 0$. Therefore, if K_{du} and K_{pu} are selected properly, then the equivalent inputs V_u can drive Harbour crane swings q_u toward zero. Therefore, the nonlinear controller asymptotically stabilizes the system state profiles. To stabilize the unactuated and actuated states, all equivalent inputs are obtained by a linear combination of V_a and V_u as:

$$V = V_a + \alpha V_u \tag{12}$$

where V_a and V_u are obtained, respectively, by eq.8 and eq.10. α is a weighting matrix. The control law U (that is the system given by the generalized forces applied by the actuators) is obtained solving the actuated dynamics (that is by equation 2).

The stability of the remaining part (the unactuated dynamics) of the closed-loop system, called the Internal dynamics, will be analyzed now. If the Internal dynamics is stable, then the asymptotic control problem is solved.

Considering eq. 12, we can obtain by eq. 5 the following relation:

$$\ddot{q}_u = -M_{22}^{-1}(q)(-M_{21}(q)K_{da} + C_{21}(q,\dot{q}))\dot{q}_a + (-M_{21}(q)\alpha K_{du} + C_{22}(q,\dot{q}))\dot{q}_u + \\ - M_{21}(q)K_{pa}(q_a - q_{da}) - M_{21}(q)\alpha K_{pu}q_u + G_2(q) \quad (14)$$

As before defined, the local stability of the Internal dynamics is guaranteed if the zero dynamics is exponentially stable. If we set, at the end of the movement, $q_a = q_{ta}$ in the internal dynamics eq.14 the zero dynamics of the system is obtained in the following way:

$$\ddot{q}_u + M_{22}^{-1}(q)\{(-M_{21}(q)\alpha K_{du} + C_{22}(q,\dot{q}))\dot{q}_u - M_{21}(q)\alpha K_{pu}q_u + G_2(q)\} = 0 \quad (15)$$

The dynamics relative to the end of the movement (zero dynamics) is expanded into a set of (nâ€“m) second-order nonlinear differential equations in which the (nâ€“m) components of the vector q_u are considered as variables.

Now, in the definition and study of the zero dynamic, we refer to [15] and to [16] .

The stability of the zero dynamics eq. 15 is analyzed using Lyapunov's linearization theorem. By defining 2(n-m) state variables $z \in R^{2x(n-m)}$, the zero dynamics eq. 15 is converted in a state form as follows:

$$\dot{z} = f(z) \quad (16)$$

where $f(z)$ is a vector of nonlinear functions, and $z \in R^{2x(n-m)}$ is a state vector. System dynamics eq. 16 is composed of 2(n-m) first-order nonlinear differential equations. This nonlinear zero dynamics is asymptotically stable around the equilibrium point $z = 0 \quad (q_u = \dot{q}_u = 0)$ if the corresponding linearized system is strictly stable. If we linearize the zero dynamics around $z = 0$ then we obtain a linearized system in the following form:

$$\dot{z} = A(z) \quad (17)$$

where

$$A = (\frac{\partial f}{\partial z})_{z=0} \quad (18)$$

is a 2(*n-m*)*x*2(*n-m*) Jacobian matrix of components $\partial f_i / \partial x_j$. The stability of the linear system eq. 17 can be analyzed by considering the positions of the eigenvalues of A or using several traditional techniques, such as the Routh-Hurwitz criterion, the root locus method, and so on. Thus, by investigating the stability of the linear system 17, we can understand the dynamic behavior of the nonlinear system eq. 16, or equivalently, zero dynamics eq. 15, according to linearization theorem of Lyapunov.

Therefore, we can summarize the stability criteria with the following statements:

1) The nonlinear system 15 is asymptotically stable around the equilibrium point if the linearized system 17 is strictly stable.
2) The equilibrium point of the nonlinear system 15 is unstable if the linearized system 17 is unstable.
3) The stability of the nonlinear system 15 cannot defined if the linearized system 17 is marginally stable.

4 Application of the Stability Analysis to an harbour crane system

4.1 Equation system and Matrices for an harbour crane

We now pass to the application of the theory of stability, described in the section, to the system given by the equations relating to the dynamics of the harbor crane.

Considering the five Lagrangian coordinates q_i described in Section 2, we derive the Lagrange function:

$$\mathsf{L} = T - V \quad (19)$$

where T is the Kinetic Energy of the crane system and V is the Potential Energy. As consequence, we apply the generalized Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i \tag{20}$$

and so we can obtain the system of equations in the generalized coordinates q_i .

In the Lagrange equations (20), the generalized non-conservative Forces Q_i are introduced relative to the dissipative forces. Particularly they represent the components, on the axes x and y, of air resistance forces acting on both the payload and boom system, the components of the wind force, the components of the forces due to the damping of the rotation movement.

The Kinetic energy of the harbour crane is the sum of the corresponding terms for the rigid tower T_T , for the boom T_B and for the payload T_L . Therefore, the Kinetic energy T of the crane system is given by

$$T = T_T + T_B + T_L \tag{21}$$

We focus our interest relative to the Lagrangian coordinates r , φ , l φ_r , φ_t , since φ , r and l are directly controlled by the generated profiles that the Plc sent to the axes drives. Assuming the small angles approximation, from Lagrange equations (20) we obtain the system of the following five equations, with three actuators used to stabilize five outputs:

$$\begin{aligned}F_r &= (m_B+m_L)\ddot{r}-m_L l\varphi_t\ddot{\varphi}+m_L\varphi_r\ddot{l}+m_L l\ddot{\varphi}_r+k_r\dot{r}\\ &-[m_B r+m_L(r+l\varphi_r)^2]\dot{\varphi}^2-2m_L\varphi_t\dot{l}\dot{\varphi}+2m_L\dot{l}\dot{\varphi}_r\end{aligned} \tag{22}$$

$$\begin{aligned}F_\varphi &= -m_L l\varphi_t\ddot{r}+J_B\ddot{\varphi}+m_L r\varphi_t\ddot{l}+m_L lr\ddot{\varphi}_t+k_\varphi\dot{\varphi}\\ &+2m_L l\varphi_r\dot{r}\dot{\varphi}+2m_B r\dot{r}\dot{\varphi}+2m_L lr\dot{\varphi}\dot{\varphi}_r+2m_L r\dot{l}\dot{\varphi}_t\end{aligned} \tag{23}$$

$$\begin{aligned}F_l &= m_L\varphi_r\ddot{r}+m_L r\varphi_t\ddot{\varphi}+m_L\ddot{l}+k_l\dot{l}\\ &-m_L r\varphi_r\dot{\varphi}^2+2m_L\varphi_t\dot{r}\dot{\varphi}-m_L g\end{aligned} \tag{24}$$

$$\begin{aligned}0 &= m_L\ddot{r}-m_L l\varphi_t\ddot{\varphi}+m_L l\ddot{\varphi}_r+k_{\varphi_r}\dot{\varphi}_r-m_L(r+l\varphi_r)\varphi^2\\ &-2m_L\varphi_t\dot{l}\dot{\varphi}-m_L(\varphi_r/l)\dot{l}^2+2m_L\dot{l}\dot{\varphi}_r-2m_L l\dot{\varphi}\dot{\varphi}_t+m_L\varphi_r g\end{aligned} \tag{25}$$

$$\begin{aligned}0 &= m_L(r+l\varphi_r)\ddot{\varphi}+m_L l\ddot{\varphi}_t+k_{\varphi_t}\dot{\varphi}_t-m_L l\varphi_t\varphi^2+2m_L\dot{r}\dot{\varphi}\\ &+2m_L\varphi_r\dot{l}\dot{\varphi}-m_L(\varphi_t/l)\dot{l}^2+2m_L\dot{l}\dot{\varphi}_t+2m_L l\dot{\varphi}\dot{\varphi}_r+m_L\varphi_t g\end{aligned} \tag{26}$$

where:

m_L and m_B represent respectively the payload and the Boom mass;

k_r , k_φ , k_l , k_{φ_r} and k_{φ_t} represent respectively the damping parameters relative to the Lagrangian coordinates;

J_B represent the inertia momentum of the tower and the boom around the z axis.

F_r , F_φ and F_l represent respectively the generalized forces generated by the three actuators;

g is the gravity acceleration.

Harbour crane dynamics can be represented by matrix equation 1 in which the component matrices are determined in the following way:

$$M(q)=\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & 0 \\ m_{21} & m_{22} & m_{23} & 0 & m_{25} \\ m_{31} & m_{32} & m_{33} & 0 & 0 \\ m_{41} & m_{42} & 0 & m_{44} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} \end{bmatrix}; \quad C(q,\dot{q})=\begin{bmatrix} k_r & c_{12} & c_{13} & c_{14} & 0 \\ c_{21} & k_\varphi & 0 & c_{24} & c_{25} \\ c_{31} & c_{32} & k_l & 0 & 0 \\ 0 & c_{42} & c_{43} & c_{44} & c_{45} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix};$$

$$F(q)=\lfloor F_r \quad F_\varphi \quad F_l \quad 0 \quad 0 \rfloor; \quad G(q)=\lfloor 0 \quad 0 \quad g_3 \quad g_4 \quad g_5 \rfloor.$$

The elements of the $M(q)$ matrix are the following:

$m_{11}=m_B+m_L;\quad m_{12}=-m_L l\varphi_t;\quad m_{13}=m_L\varphi_r;\quad m_{14}=m_L l;$

$m_{21}=-m_L l\varphi_t;\quad m_{22}=J_H;\quad m_{23}=m_L r\varphi_t;\quad m_{25}=m_L rl;$

$m_{31}=m_L\varphi_r;\quad m_{32}=m_L r\varphi_t;\quad m_{33}=m_L;$

$m_{41}=m_L;\quad m_{42}=-m_L l\varphi_t;\quad m_{44}=m_L l;\ m_{52}=m_L(r+l\varphi_r);\quad m_{55}=m_L l.$

The elements of the $C(q,\dot{q})$ matrix are the following:

$c_{12}=-[m_B r+m_L(r+l\varphi_r)^2]\dot{\varphi};\quad c_{13}=-2m_l\varphi_t\dot{\varphi};\quad c_{14}=2m_L\dot{l};$

$c_{21}=2m_L(m_l+m_B)(l+r)\dot{\varphi};\quad c_{24}=2m_L lr\dot{\varphi};\quad c_{25}=2m_L r\dot{l};\qquad c_{31}=2m_L\varphi_t\dot{\varphi};\quad c_{32}=-m_L r\varphi_r\dot{\varphi};$

$c_{42}=-m_L(r+l\varphi_r)\dot{\varphi};\quad c_{43}=-m_L(2\varphi_t\dot{\varphi}+\varphi_r\dot{l}/l);\quad c_{44}=2m_L\dot{l}+k_r;\quad c_{45}=-2m_L l\dot{\varphi};$

$c_{51}=2m_L\dot{\varphi};\quad c_{52}=m_L(-l\varphi_t\dot{\varphi}+2\varphi_r\dot{l});\quad c_{53}=-m_L\varphi_t\dot{l}/l);\quad c_{54}=2m_L l\dot{\varphi}; c_{55}=2m_L\dot{l}+k_t.$

At the end, the elements of the $G(q)$ vector are defined as:

$g_3=-m_L g;\quad g_4=m_L\varphi_r g;\quad g_5=m_L\varphi_t g.$

4.2 Stability Analysis for an harbour crane

Applying the theory proposed in Section 3, we analyze the local stability of the internal dynamics eq. 14, or equivalently, the zero dynamics eq. 15. We define the control of the system starting by equations 9 and 11, where $K_{da} = diag(K_{da1},K_{da2},K_{da3})$, $K_{pa} = diag(K_{pa1},K_{pa2},K_{pa3})$, $K_{du} = diag(K_{du1},K_{du2})$, $K_{pu} = diag(K_{pu1},K_{pu2})$ are the positive matrices of the control gains and $\alpha=\begin{bmatrix}\alpha_1 & 0 \\ 0 & \alpha_2 \\ 0 & 0\end{bmatrix}$ is a weighting matrix. Therefore, applying matrix equation 15 to the equations of a harbour crane described by the equations from 22 to 26, and developing the matrices defined in eq. 15, the zero dynamics of the system is obtained as:

$$0=\ddot{\varphi}_r+(\frac{1}{l})\{[2\dot{l}+(\frac{k_r}{m_L})-\alpha_1 K_{du1}]\dot{\varphi}_r+(-2l\dot{\varphi}+l\varphi_t\alpha_2 K_{du2})\dot{\varphi}_t$$
$$-\varphi_r\alpha_1 K_{pu1}+l\varphi_t^2\alpha_2 K_{pu2}+g\varphi_r\} \tag{27}$$

$$0=\ddot{\varphi}_t+(\frac{1}{l})\{(2l\dot{\varphi})\dot{\varphi}_r+[2\dot{l}+(\frac{k_t}{m_L})-(r+l\varphi_r)\alpha_2 K_{du2}]\dot{\varphi}_t+$$
$$-[(r+l\varphi_r)\varphi_t\alpha_2 K_{pu2}]+g\varphi_t\} \tag{28}$$

The stability of the zero dynamics is analyzed using linearization theorem of Lyapunov. Therefore, we set the four state variables as following:

$$z_1 = \varphi_r; \quad z_2 = \varphi_t; \quad z_3 = \dot{\varphi}_r; \quad z_4 = \dot{\varphi}_t;$$

From equations 27 and 28 we obtain the following system:

$$\dot{z}_1 = z_3 \tag{29}$$

$$\dot{z}_2 = z_4 \tag{30}$$

$$\dot{z}_3 = -(\frac{1}{l})\{[2\dot{l} + (\frac{k_r}{m_L}) - \alpha_1 K_{du1}]z_3 + (-2l\dot{\varphi} + lz_2\alpha_2 K_{du2})z_4 - z_1\alpha_1 K_{pu1} + lz_2^2\alpha_2 K_{pu2} + gz_1\} = f_1(z) \tag{31}$$

$$\dot{z}_4 = -(\frac{1}{l})\{(2l\dot{\varphi})z_3 + [2\dot{l} + (\frac{k_t}{m_L}) - (r + lz_1)\alpha_2 K_{du2}]z_4 + -[(r + lz_1)z_2\alpha_2 K_{pu2}] + gz_2\} = f_2(z) \tag{32}$$

Considering the state vector $z = [z_1 \quad z_2 \quad z_3 \quad z_4]$, the nonlinear zero dynamics described by equations by 29 to 32, are asymptotically stable around the equilibrium point $z = 0$ if the linearized system is strictly stable. That involves a linear system (around $z = 0$) as follows:

$$\dot{z} = Az \tag{33}$$

being

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial z_2} & \frac{\partial f_1}{\partial z_3} & \frac{\partial f_1}{\partial z_4} \\ \frac{\partial f_2}{\partial z_1} & \frac{\partial f_2}{\partial z_2} & \frac{\partial f_2}{\partial z_3} & \frac{\partial f_2}{\partial z_4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{(\alpha_2 K_{pu2}) - gz}{l} & 0 & \frac{\alpha_1 K_{du1} - 2\dot{l} - (\frac{k_r}{m_L})}{l} & 0 \\ 0 & \frac{r(\alpha_2 K_{pu2}) - g}{l} & -2\dot{\varphi} & \frac{r(\alpha_2 K_{du2})}{l} \end{bmatrix} \tag{34}$$

A is a Jacobian matrix. It follows that the characteristic polynomial has the form:

$$det(A - sI\ 4) = s^4 - s^3\{\frac{\alpha_1 K_{du1} + r\alpha_2 K_{du2} - [2\dot{l} + (\frac{k_r}{m_L})]}{l}\} - s^2(\frac{\alpha_1 K_{pu1} + r\alpha_2 K_{pu2} - g}{l}) + s\{\frac{[\alpha_1 K_{du1} - 2\dot{l} - (\frac{k_r}{m_L})](r\alpha_2 K_{pu2} - g) + (\alpha_1 K_{pu1} - g)(r\alpha_2 K_{du2})}{l^2}\} + \frac{(\alpha_1 K_{pu1} - g)}{l^2} \tag{35}$$

The linearized system 33 is stable around the equilibrium point $z = 0$ if the matrix A is a Hurwitz matrix. Applying the Hurwitz's criterion (see [15]) and considering the result of the calculations, we come to the final goal of our work.

In fact, as consequence of eq. 35, we can establish the constraint conditions for the controller parameters which result necessary to define the stability of the system. They are:

$$\alpha_1 K_{du1} + r\alpha_2 K_{du2} < [2\dot{l} + (\frac{k_r}{m_L})] \tag{36}$$

$$\alpha_1 K_{pu1} + r\alpha_2 K_{pu2} < g \tag{37}$$

$$[\alpha_1 K_{du1} - 2\dot{l} - (\frac{k_r}{m_L})](r\alpha_2 K_{pu2} - g) + (\alpha_1 K_{pu1} - g)(r\alpha_2 K_{du2}) > 0 \tag{38}$$

$$\alpha_1 K_{pu1} > g \tag{39}$$

As we see from eq. 37 and eq. 39, the α_2 parameter must be negative.

5 Conclusion

In this work, we investigate the stability of a model for controlling the Sway of a Harbour crane. The set of nonlinear differential equations describing the Harbour crane dynamics is obtained and the system is divided into two subsystems: the first one for actuated outputs and the second one for unactuated outputs. The control system is defined by linearly combining two components that are separately obtained from the nonlinear feedback of actuated and unactuated states. Hurwitz's criterion was applied to investigate the system stability for the equations describing the movement control of the harbour crane. We reach the goal to establish the constraint conditions which result necessary to define the stability of the system describing the harbour crane dynamics.

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