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Fractal Image Coding with Combination of Wavelet Pretreatment and Compressed Sensing

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Abstract: Classical fractal image coding can achieve high compression ratio, but it is also accompanied with complex algorithm and long encoding time. However, combining other coding techniques with the classical fractal image coding can effectively shorten the encoding time. Here, based on the combination of the compressed sensing technology and the wavelet transform technology, a fractal image coding technology combined with the wavelet pretreatment and compressed sensing is proposed. **Method:** This method firstly extracts the high frequency and low frequency signals components from the original image with wavelet transform, which includes discrete first-order, second-order and third-order wavelet transform. Then, the compressed sensing coding is applied to the high frequency component while the fractal theory encoding is used to process the low frequency component. Thus, the image can be effectively restored. Finally, a comparison of the proposed technology and the classical fractal image coding has been discussed on the aspects of Encoding Time, Compression Ratio(CR) and Peak Signal to Noise Ratio (PNSR). **Result：** The proposed technology has been demonstrated with ten images. It is found that by combining the first-order wavelet pretreatment to the compressed sensing and the fractal image coding, the PSNR index of the restored image is closed to that obtained from the classical fractal image coding. The compression ratio is much larger and the coding time decrease with 6 times. In the case of the second-order wavelet pretreatment is applied to the combination of compressed sensing and fractal image coding, the PSNR of the restored image decreases slightly as compared to that of the classical fractal image. The coding time continue to decrease. Even though the compression ratio decreases, it is also higher than that of the classical fractal image coding. The third-order wavelet pretreatment combined compressed sensing with fractal image coding is also studied. The results show that the PSNR of the restored image is lower than that of the classical fractal image coding, as well as the compression ratio. However, the coding speed is the fastest as compared to the two cased discussed previously in this paper. Therefore, it can be conclude that as the order of the wavelet transform increases, the average of the coding time is greatly saved although the average of the compression rate drops while the average of the PSNR of the restored image is only slightly reduced. The coding time were reduced by an average of 6.28 times, 24.98 times, 124.70 times in the cases of the discrete first-order wavelet transform, discrete second-order wavelet transform and discrete third-order wavelet transform pretreatment combined compressed sensing with fractal image coding. **Conclusion:** The proposed technology in this paper can not only maintain the high quality of restored images, but also effectively shorten the coding time than that of the classical fractal image coding. The proposed technology can be widely applied in the procession of image and video compression.
Keywords: fractal image coding; Compressed Sensing; Wavelet Transform; Encoding Time

1. Introduction

With the arrival of cloud computing and large data style, a lot of data needs to be stored. It is particularly important to study image compression coding. In 1998, some people will be based on fractal compression technology used in image coding, the iterative function system[1]. Fractal image coding can achieve 10000: 1 for image compression with good self-similarity[2]. Fractal image coding compression rate is high, but its coding time is longer. How to shorten the encoding time of fractal image coding and improve the image recovery effect, many scholars put forward the improved method[3-4]. Such as fast fractal image encoding of optional features[5]. A Fast Fractal Image Coding Algorithm for 1-norm Matching[6]. Fast fractal image coding based on image block geometric segmentation[7]. In order to take full advantage of different coding techniques, fractal image coding combined other coding. Based on wavelet transform with fractal image coding[9-11], Based on discrete cosine transform with fractal image coding[12]. And combined compressed sensing with fractal image coding[13]. The focus of this paper is the wavelet pretreatment combined compressed sensing with fractal image coding. Different from the literature[13]. This paper studies based on the combination of the compressed sensing technology and the wavelet transform technology of compression ratio index, PSNR index and time index, Which are based on different wavelet transform times. Compressed sensing is proposed by scientists such as EJ Candes, J. Romberg, T. Tao and DL Donoho in 2004. Compressed sensing theory does not need to meet the requirements of the Nyquist sampling theorem, only to satisfy the sampling frequency More than the signal frequency, the original signal can be almost no loss of recovery[14-16]. In this paper, the discrete wavelet transform is used to process the image, and the high frequency signal and the low frequency signal of the image are obtained respectively. The high frequency signal of the image is encoded according to the compression technology, and the low frequency technology of the image is encoded according to the classical fractal image coding technique. Will combine the two to restore the image. In the first section of the article, the development of fractal image coding and compression sensing technology is introduced. The second section introduces the classical fractal coding technology and compressed sensing coding technology. The third section is the experiment and discussion, and the difference between the box dimension and the whole variance of the image is used to reflect the image self-similarity. The different levels of discrete wavelet transform are used to preprocess the image. A comparison of the effects of wavelet pretreatment combined compressed sensing with fractal image coding and other coding techniques is discussed. The fourth section is the summary of the article, the Flowchart of the proposed wavelet pretreatment combined compressed sensing with fractal image encoding and decoding shown in Figure 1:

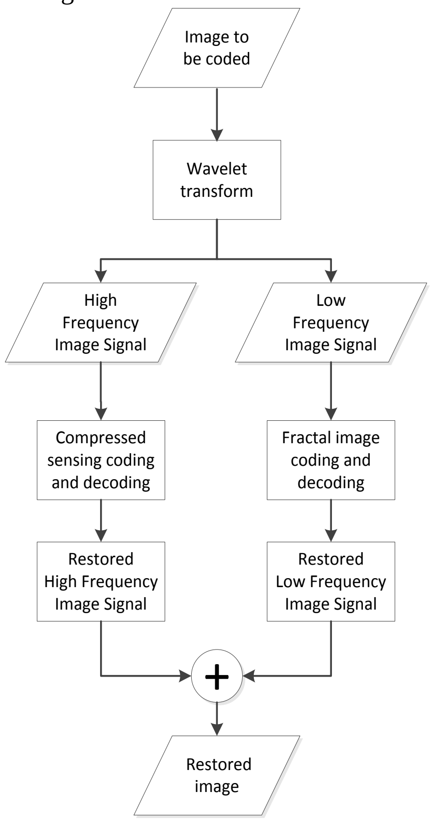

Fig.1 Flowchart of the proposed wavelet pretreatment combined compressed sensing with fractal image encoding and decoding.

As shown in Fig.1, the image is subjected to two-dimensional discrete wavelet transform to obtain the image high frequency signal and the image low frequency signal. Then the lowfrequency signal of the image is encoding according to the classical fractal image coding, and the high frequency signal of the image is encoding according to the compression sensing technology. Finally, combined two methods to get a recovery images.

2. Classical fractal coding and compressed sensing coding algorithm

2.1 Fractal image coding of low frequency images

The block diagram of the fractal image is shown below[17]

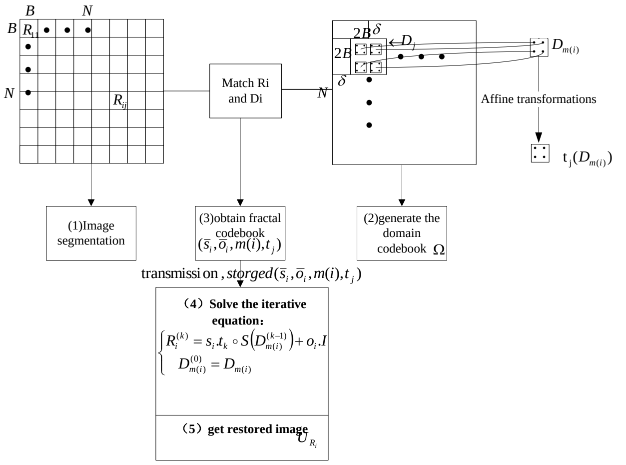

Fig2. Block diagram of the classical fractal image encoding and decoding.

As shown in Fig 2.(1) segmentation the image: First, the size of the N*N low-frequency image is divided into the size of B*B image block, and then generate the corresponding range block R, their sum is the image to be encoded.

(2) generate codebook Ω: In the original image in accordance with the step size δ from left to right, from top to bottom to move a size of 2B*2B D block pool. Then, each block D is compressed by the 2*2 Area mean to obtain a block of pixels of size B*B. These pixel blocks generate the codebook Ω as a whole.

(3)Output the fractal codebook: For each $D \in \Omega$, first in accordance with the literature [17] in the formula 3.19 to find the contrast factor s_i , quantization factor o_i , its quantitative and sum $\bar{s}_i$ and $\bar{o}_i$, and then [17] in the 3.20 corresponding to the most matching codebook. Finally, in accordance with the formula (1) constraints in the codebook to find the best match the remain of the domain block.

$$E(R_i, D_{m(i)}) = \min_{D\in\Omega} E(R_i, D) \quad (1)$$

(4)Which requires eight affine transformations to obtain eight sub-blocks, according to formula (2) to find the error of the smallest isometric type.

$$E(R_i, t_j(D_{m(i)})) = \min_{0\le j\le 7} E(R_i, t_j(D_{m(i)})) \quad (2)$$

Finally get the R_i best approximation

$$s_i.t_k(D_{m(i)}) + o_i.I \quad (3)$$

Need to encode each block R_i , the output of the quantized after the fractal code $\left(\bar{s}_i, \bar{o}_i, m(i), t_j\right)$, get fractal codebook.

(5) Fractal image decoding: The essence of fractal image decoding is the iterative equation[17]:

$$\begin{cases} R_i^{(k)} = s_i \cdot t_k \circ S\left(D_{m(i)}^{(k-1)}\right) + o_i \cdot I \\ D_{m(i)}^{(0)} = D_{m(i)} \end{cases} \quad (4)$$

2.2 Compressed sensing coding of the high frequency images

The compressed sensing theory framework is as follows:

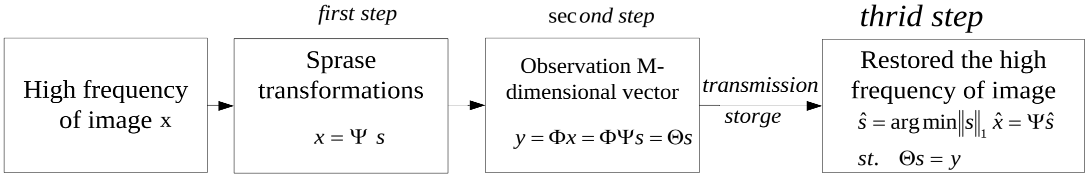

Fig.3 Theory framework of CS

The image coding framework of compressed sensing can be defined as:

$$x = \psi s \quad (5)$$

$$y_k = \langle x, \phi_k \rangle, k = 1,2,......M \quad (6)$$

Where ϕ_k is the observation matrix of M*N and ψ is a spares matrix of M*N. We assume that M is much smaller than N. As for 0-norm（the number of nonzero value） $\|s\|_0 \le k$, the compressed sensing (CS) equation is $y = \Phi x = \Phi \Psi s = \Theta s$, where $\Theta = \Phi \Psi$ is the recovery matrix.

Since signals are sparse themselves, we only need to find out s. The constraint equation of signal reconstruction is:

$$\hat{s} = \arg\min \|s\|_1 \quad s.t. \quad \Theta s = y \qquad (7)$$

Solving the approximation value $\hat{\alpha}$ of α, we get the reconstruction signal $\hat{x} = \Psi \hat{\alpha}$.

In reality, the system may be affected by noise polution. Therefore we need to add noise cancellation to the constraint equation:

$$\hat{s} = \arg\min \|s\|_1 \quad s.t. |\Theta s - y| \le \varepsilon \qquad (8)$$

Where ε is the maximum of noise.

In our research, we use OMP for solve the compressed sensing algorithm:

Source signal x: Enter the high frequency signal of the image

Recovery signal $\hat{x}$: Restored the high frequency signal of the image

The description of the compressed sensing coding is as follows:

Step 1: the high-frequency part of the image processing; which is K sparse, there are K non-zero coefficient.

Step 2: Update the index set $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$, in the sensor matrix to rebuild the atomic set $\Phi_t = [\Phi_{t-1}, \Theta_j]$;

Step 3: Least square method is obtained $\hat{x} = \arg\min \|y - \Phi_t \hat{x}\|_2$;

Step 4: Update the residuals $r_t = y - \Phi_t \hat{x}_t, t = t+1$

Step 5: Determine whether to meet $t > k$, if satisfied, then stop iteration; if not satisfied, then repeat step 1.

3. Experiment and discussion

The experimental data used in this study are images of 256 * 256 with ten pixels as shown in Fig. 4, corresponding to 1-10 (11 for 10 images) in Fig. 5a-c, respectively. The ten images are from the database USC-SIPI.

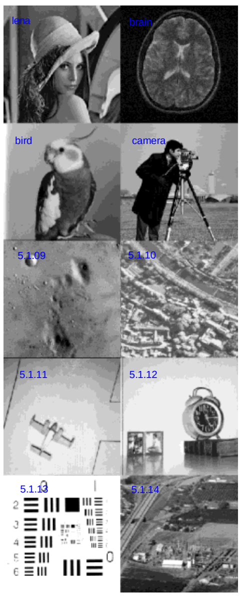

Fig.4 Images for the experimental demonstrations.

The experimental platform is a Dell workstation with 24GB memory and Intel (R) Xeon (R) CPU (X5657 / 3.06GHz). Using the programming language for the 2012 version of Matlab. The image restoration

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evaluation index mainly includes compression ratio CR, peak signal-to-noise ratio (PSNR) and encoding time. The compression ratio is defined as the ratio of the amount of data compressed in the same graph to the amount of data before compression. Usually the higher of the compression ratio, the smaller of the storage space it needs. PSNR higher image recovery effect is better. The shorter of the encoding time, the higher of the coding efficiency.

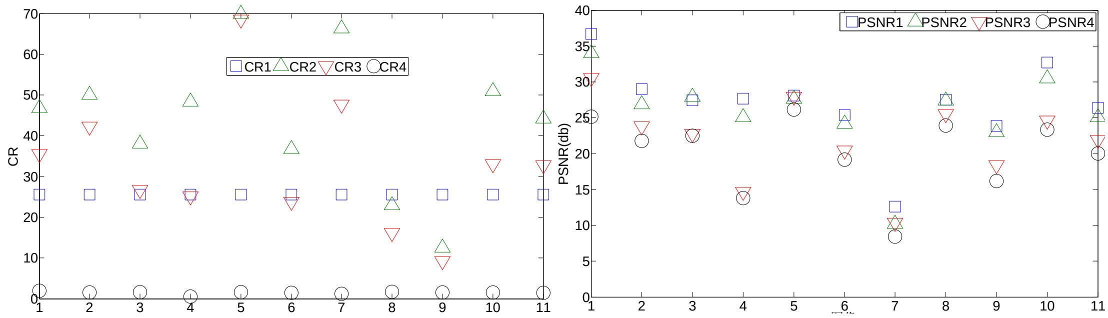
In this paper, we study the complexity of the image texture by calculating the difference box dimension of the 10 images. At the same time, the self-similarity of the whole image is studied by calculating the variance of the whole image. The experimental results are shown in Table 1:

Table 1 Self-similarity of the images

Grayscale image	Differential box dimension	Image standard deviation
bird.jpg	2.2865	49.5392
brain.bmp	2.5246	52.2920
camera.jpg	2.4600	36.8856
lena.bmp	2.4549	62.3452
5.1.09.tiff	2.4320	27.7362
5.1.10.tiff	2.6036	45.342
5.1.11.tiff	2.2923	33.0725
5.1.12.tiff	2.3915	57.2497
5.1.13.tiff	2.5126	75.7531
5.1.14.tiff	2.5082	42.3579

As shown in Table 1, the most complex of image texture is the 5.1.10.tiff image, the easiest of image texture is namely bird.jpg. bird.jpg image with the classic fractal image encoding has the best recovery effect. The overall self-similarity of the image is 5.1.09.tiff image, the weakest self-similarity is 5.1.13.tiff. These images have some self-similarity.

In this paper, the above 10 images are subjected to wavelet pretreatment combined compressed sensing with fractal image coding. The two-dimensional discrete first-order wavelet transform, two-dimensional discrete second-order wavelet transform and two-dimensional discrete third-order wavelet transform are obtained for different images respectively, and the corresponding high and low frequency signals are obtained. The low frequency signal of the image includes the basic information of the image, and the high frequency signal of the image includes the detail part of the image. The low frequency signal of the image is coding by the classical fractal algorithm, and the high frequency signal of the image by the compressed sensing coding. The relationship between the CR, the image PSNR and the compression ratio of the image is compared with the classical fractal image coding under different wavelet transform levels. In this paper, the sampling value of the compression technology is 190, and the classical fractal image is B = 4. The results of wavelet preprocessing combined compressed sensing with fractal image coding are shown in Figure 5.a-c:



A Compression rate of coded image with different fractal images

b PSNR of Different Fractal Image Coded Images

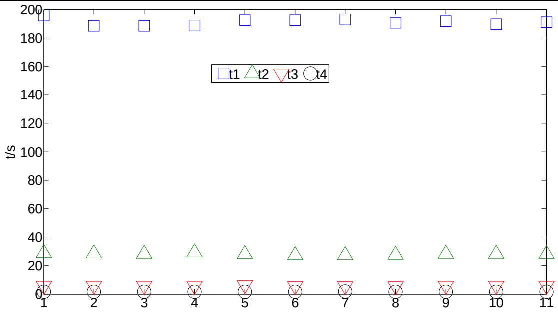


Fig.5 Compression of recovery image indexes of different fractal image coding

CR1, PSNR1, t1 represent the compression ratio, peak singal to noise ratio, and coding time, respectively, using classical fractal image coding

CR2, PSNR2, t2 represent the compression ratio, peak signal to noise ratio, and coding time, respectively, using fractal image coding based on Discrete first-order wavelet transform combined compressed sensing with fratcal image coding

CR3, PSNR3, and t3 represent the compression ratio, peak signal-to-noise ratio, and coding time obtained by fractal image coding based on discrete second-order wavelet transform combined compressed sensing with fractal image coding

CR4, PSNR4, t4 represent the compression ratio, peak signal-to-noise ratio, and coding time, which are obtained by fractal image coding based on discrete third-order wavelet transform combined compressed sensing with fractal image coding

Figure 5a-c shows that the average compressed ratio and the average PSNR are based on the first-order wavelet transform combined compressed sensing with fractal image coding, but the coding time is the longest. the number of wavelet transform increase, the average compression ratio and PSNR decreased, fortunately, the coding time is also reduced.

Based on the first-order wavelet transform combined compressed sensing with fractal image coding to maintain the classic fractal image coding PSNR, and CR than its larger, coding time shorter than the 6.28 times. The coding time of fractal image coding based on second-order wavelet transform combined compressed sensing with fractal image coding perception is 24.98 times shorter than that of classical fractal image coding. The average coding time of fractal image coding based on three-level wavelet transform combined compressed sensing with fractal image coding perception is 124.70 times shorter than that of classical fractal image coding. The more the series of discrete wavelet transforms, the smaller the low frequency signal pixels of the image. Because the image of the low frequency signal using the classic fractal image coding. So the smaller the image, the shorter the coding time required. And the image of the high-frequency signal and the size of the image to be encoded the same, the image of the high-frequency signal compression coding technology encoding and decoding, high-frequency image coding time is basically the same. Therefore, the coding time of wavelet preprocessing combined compressed-sensing with fractal image coding technology will decrease with the increase of the number of wavelet transforms.

The following is a combination of different wavelet transform times combined compressed sensing with fractal image coding recovery effect, here only have bird.jpg. as an example. The experimental results are shown in Figure 6.a-c:

a b c

Fig.6 The restored images using different wavelet pretreatment combined compressed sensing with fractal image coding.

a image recovery effect based on first-orderwavelet transform combined compression sensingwith fractal image coding

b image recovery effect based on second-order wavelet transform combined compressed sensing with fractal image coding

c image recovery effectbased on three-level wavelet transform combined compressed sensing withfractal image coding

According to Figure 6.a-c, the image recovery effect based on the first-order wavelet transform combined compressed sensing with fractal image coding is the best. Based on the second-order wavelet transform combined compressed sensing with fractal image coding, the image restoration effect is the second,Based on the third-order wavelet transform combined compressed sensing with fractal image coding combining compressed sensing with fractal image coding is worst.

4. Conclusion

In this paper, the experimental results show that the wavelet preprocessing combined compressed sensing with fractal image coding maintains the PSNR index of classical fractal image coding, Which greatly shortens the coding time and improves the compressed ratio. If the recovery image requires a high compressed ratio and the recovery effect is better, the coding time is much smaller than the classic fractal image coding. Fractal image coding based on first-order wavelet transform combined compressed sensing with fractal image coding can be selected. If you need good recovery effect, coding time is shorter, you can choose based on three-level wavelet transform combined compression sensing with fractal image coding.

You can also use this method to study images of other pixels, or color images. In the image transmission process to reduce the amount of storage, improve the image recovery effect. Future is expected to be used in video processing. Image pretreatment can also be performed using other wavelet transform. Or select other fractal image coding and compression sensing coding.

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