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Transient Analysis of Three-Phase Wound Rotor (Slip-Ring) Induction Motor under Operating Condition

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Abstract: This work investigates the transient behaviours of a three phase wound rotor type induction motor, running on load. When starting a motor under load condition becomes paramount, obviously a wound rotor (slip ring) induction becomes the best choice of A.C. motor. This is because maximum torque at starting can be achieved by adding external resistance to the rotor circuit through slip-rings. Normally a face-plate type starter is used, and as the resistance is gradually reduced, the motor characteristics at each stage changes from the other (John Bird 2010). Hence, the three phase wound rotor (slip ring) induction motor competes favourably with the squirrel cage induction motor counterpart as the most widely used three-phase induction motor for industrial applications. The world-wide popularity and availability of this motor type has attracted a deep – look and in-depth research including its transient behavior under plugging (operating) condition. This paper unveils, through mathematical modeling, followed by dynamic simulation, the transient performance of this peculiar machine, analyzed based on Park's transformation technique. That is with direct-quadrature-zero (d-q-o) axis based modeling in stationary reference frame.

Key Words: Transient Condition, d-q-0 reference frame, a-b-c reference frame, arbitrary reference frame, stationary reference frame, and wound rotor induction motor etc.

Introduction

When a circuit possess energy-storing element such as inductance and capacitance, the energy state of the circuit can be disturbed by changing the position of the switch connecting the elements to the source. The circuit then settles down to another steady state after a certain time. (Smarajit Ghosh 2011). Interestingly, the behavior of all electrical machines is only of interest when it is in a steady-state. There exist some cases, however, when the temporary response of a machine to a change in conditions is required. For instance, if a power supply in a machine is switched on, there may be a surge, possibly with oscillation before a steady flow of current is established. This surge (burst of energy) is short-lived and instantnous, and is caused by a sudden change of state of the machine.

Circuits exhibit transient when they contain active components, that can store energy, such as transformers and induction motors. Transient phenomenon is an aperiodic function. The duration for which it lasts is very insignificant as compared with the operating time of the system.

Yet they are very important because, depending upon the severity of these transients, the system may result into shut down of a plant etc. (Wadjwa 2009).

Every electrical system can be considered as made up of linear impedance of elements resistance, inductance and capacitance. For the slip-ring induction motor under discussion, its circuit is normally energized and carries mechanical load, until a fault suddenly occurs. The fault, then corresponds to the closing of a switch (or switches, depending upon the type of fault). The closing of this/these switch(es) charge(s) the circuit, so that a new distribution of currents and voltages is brought about. This redistribution is accompanied in general by a transient period during which the resultant currents and voltages may momentarily be relatively high. It pays to realize that this redistribution of currents and voltages cannot take place instantaneously for the following reason:

a) The electromagnetic energy stored by an inductance (L)(main component of every electrical machine) is $0.5\ LI^2$, where I is the instantaneous value of current. Assuming inductance to be constant, the change in magnetic energy requires change in current which an inductor is opposed by an e.m.f of magnitude $L\ \frac{dI}{dt}$. In order to change the current instatenousely dt = 0 and therefore $L\ \frac{dI}{0} = \infty$ (infinity). This suggests that to bring about instantaneous change in current the e.m.f. in the inductor should become infinity which is practically impossible. Hence the charge of energy in an inductor is gradual, and therefore the redistribution of energy following a circuit change takes a finite time.

b) Another component, the resistance R (also obtainable at the stator/rotor windings) consumes energy. At any time, the principle of energy conservation in an electrical circuit applies. That is the rate of generation of energy is equal to the rate of storage of energy, plus the rate of energy consumption.
In its summarized form, it becomes clear that in any rotating machine;
i) the current cannot change instantaneously through an inductor (winding)
ii) the law of conservation of energy must hold good, are fundamental to the phenomenon of transient in electric machine.
Also from the above explanations, it can be said that in order to have transients in an electrical machine with special reference to three phase wound rotor (slip ring) induction motor, the following requirements should be met.
i) The inductor (windings) should be present
b) A sudden change in the form of a fault or any switching operation should take place.

2- THE TRANSIENT MATHEMATICAL MODELLING OF THE MOTOR USING PARK'S TRANSFORMATION TECHNIQUE

Unlike the steady state mathematical modeling of induction motors, which is not new and has received a considerable attention from researchers, the transient mathematical modeling of induction motors continues to received much attentions. This is as a result of the vital effect the transient behavior of the induction motors has on the overall performance of the system to which it forms a component part. Transient modeling of induction motors prove to be more difficult both in the definition of suitable forms of equations and in the application of appropriate numerical methods needed for the solution of same (Okoro 2003).
The three phase wound rotor (slip-ring) induction motor is modeled using Park's transformation technique (**Direct-quadrature-zero** (d-q-o) axis transformation theory).
In electrical engineering, and other associated disciplines, the analyses of three-phase circuits are mostly simplified using **d-q-o** transformation.
It is a more reliable and accurate technique for investigating the problems of voltage drips, oscillatory torques and harmonics in electrical system, which arise due to high currents drawn by all induction motors when they are started and during the other transient operations.
The three reference frames includes;
i) Stationary reference frame
ii) Rotor reference frame
iii) Synchronous reference frame

These reference frames are used to convert the **a-b-c** reference frame (input voltage) to the **d-q-o** reference frame, and/output current (**d-q-o** reference frame) to the **a-b-c** reference frame.
Any one of the afore-mentioned three reference frame can be used to analyze the transient dynamic behavior of the three phase motor under consideration, based on the following conditions;
i) If the stator voltage is unbalanced and the rotor voltages are balanced, the stationary reference frame is useful
ii) If the rotor voltages are unbalanced and stator voltages are balanced, the rotor reference frame is used
iii) If the stator and rotor voltages are balanced and continuous, then synchronous reference frame is used

2.1 – TRANSFORMATION OF a-b-c TO d-q-o REFERENCE FRAME

The transformations like **d-q-o** developed by park can facilitate the computation of the transient solution of the three-phase wound rotor induction motor model by transforming inductances to differential equations with constant inductances.

2.2- Equations of Induction Motor in Arbitrary d-q-o Reference Frame

The relationship between the abc quantities and qdo quantities of a reference frame rotating at an angular speed, w, is shown in figure 1 below;

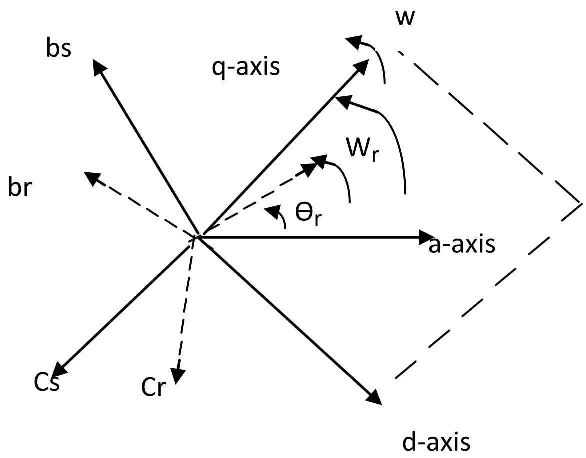


Figure 1 - Relationship between abc and arbitrary d-q-o

The transformation equation from **a-b-c** to this **d-q-o** reference frame is given by:

by: $$\begin{bmatrix} f_q \\ f_d \\ f_o \end{bmatrix} = \left[T_{qdo}\right] \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \qquad ...1$$

Where the variable **f** can be the phase voltage, currents or flux linkages of the machine. The first quadrant of the reference frame rotating at an arbitrary speed w (t), in figure 1 is shown as demarcated. The transformation angle, θ **(t),** between the **q-axis** of the reference frame rotating at w and the **a-axis** of stationary stator winding may be expressed as:

$$\theta\,(t) = \int_o^t w\,(t)dt + \theta(0) \quad elect.rad \qquad ...2$$

Likewise, the rotor angle, $\theta_r(t)$, between the axes of the stator and the rotor a-phases for a rotor rotating with speed $w_r(t)$ may be expressed as

$$\theta_r(t) = \int_o^t w_r(t)dt + \theta_r(0) \quad elect.rad \qquad ...3$$

The angles θ **(0)** and θ_r **(0)** are the initial values of those angles at the beginning of time **t**.

The qdo transformation matrix, $\left[T_{qdo}\,(\theta)\right]$ is

$$T_{qdo}(\theta) = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - 2\pi/3\right) & \cos\left(\theta + 2\pi/3\right) \\ \sin\theta & \sin\left(\theta - 2\pi/3\right) & \sin\left(\theta + 2\pi/3\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} ...4$$

And its inverse is

$$\left[T_{qdo}\right]^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - 2\pi/3\right) & \sin\left(\theta - 2\pi/3\right) & 1 \\ \cos\left(\theta + 2\pi/3\right) & \sin\left(\theta + 2\pi/3\right) & 1 \end{bmatrix} \qquad ...5$$

2.3 – d-q-o Voltage Equations

In matrix notation, the stator windings a b c voltage can be expressed as

$$V_S^{abc} = p\lambda_{abc}^s + r_S^{abc}\, i_S^{abc} \qquad …6$$

Applying the transformation to equation (6), we obtain

$$T_{qdo}(\theta)V_s^{abc}=T_{qdo}(\theta)\ p\,\lambda_{abc}^{S}+T_{qdo}(\theta)\ r_S^{abc}\,i_S^{abc} \qquad ...7a$$

$$V_S^{qdo}=T_{qdo}(\theta)\ p\left[T_{qd0}(\theta)\right]^{-1}\lambda_{qdo}^{S}+\left[T_{qdo}(\theta)\right]r_s^{abc}\left[T_{qdo}(\theta)\right]^{-1}\ i_s^{qdo} \qquad ...7b$$

$$\left[T_{qdo}(\theta)\right]^{-1}=\begin{bmatrix}\cos\theta & \sin\theta & 1\\ \cos\left(\theta-2\pi/3\right) & \sin\left(\theta-2\pi/3\right) & 1\\ \cos\left(\theta+2\pi/3\right) & \sin\left(\theta+2\pi/3\right) & 1\end{bmatrix}$$

∴

$$p\left[T_{qdo}(\theta)\right]^{-1}=w\begin{bmatrix}-\sin\theta & \cos\theta & 0\\ -\sin\left(\theta+2\pi/3\right) & \cos\left(\theta-2\pi/3\right) & 0\\ -\sin\left(\theta+2\pi/3\right) & \cos\left(\theta+2\pi/3\right) & 0\end{bmatrix}\left[\lambda_s^{qdo}\right]+\left[T_{qdo}(\theta)\right]^{-1}\ p\left[\lambda_S^{qdo}\right] \qquad ...8$$

Substituting equation (8) back into equation (7b) gives:

$$V_S^{qdo}=T_{qdo}(\theta)\,w\begin{bmatrix}-\sin\theta & \cos\theta & 0\\ -\sin\left(\theta-2\pi/3\right) & \cos\left(\theta-2\pi/3\right) & 0\\ -\sin\left(\theta+2\pi/3\right) & \cos\left(\theta+2\pi/3\right) & 0\end{bmatrix}\left[\lambda_S^{qdo}\right]+\left[T_{qdo}(\theta)\right]^{-1}\ p\left[\lambda_s^{qdo}\right]+r_s\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}i_S^{qdo} \qquad .9$$

Rearranging (9) will give:

$$V_S^{qdo}=w\begin{bmatrix}0&1&0\\-1&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}\lambda_{qs}\\ \lambda_{ds}\\ \lambda_{os}\end{bmatrix}+\ p\,\lambda_S^{qdo}+r_S^{qdo}.i_S^{qdo} \qquad ...10$$

$$r_s^{\,qdo}=\ r_S\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Where, $w=\dfrac{d\theta}{dt}$,

and

$$p=\frac{d}{dt}$$

$$\left.\begin{aligned}V_{qs}&=w\lambda_{ds}+p\,\lambda_{qs}+r_s\,i_{qs}\\ V_{ds}&=-\,w\lambda_{qs}\ +p\,\lambda_{ds}+r_s\,i_{ds}\\ V_{os}&=\ p\,\lambda_{os}+r_s\ i_{os}\end{aligned}\right\} \qquad ...11$$

2.4 – q-d-o Transformation Of The Rotor

$$v_r^{abc}\ =r_r\ i_{abc}+p\,\lambda\ _r^{abc} \qquad ...12$$

The rotor quantities must be transformed on the same **d-q-o** frame. The transformation angle to the rotor phase quantities is **(θ - θr)**. Therefore T **(θ - θr)** is the transformation angle.

$$T(\theta-\theta_r)=\frac{2}{3}\begin{bmatrix}\cos(\theta-\theta_r) & \cos\left(\theta-\theta_r-\frac{2\pi}{3}\right) & \cos\left(\theta-\theta_r+\frac{2\pi}{3}\right)\\ \sin(\theta-\theta_r) & \sin\left(\theta-\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta-\theta_r+\frac{2\pi}{3}\right)\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix} \qquad ...13$$

$$T(\theta-\theta_r)^{-1}=\begin{bmatrix}\cos(\theta-\theta_r) & \sin(\theta-\theta_r) & 1\\ \cos\left(\theta-\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta-\theta_r-\frac{2\pi}{3}\right) & 1\\ \cos\left(\theta-\theta_r+\frac{2\pi}{3}\right) & \sin\left(\theta-\theta_r+\frac{2\pi}{3}\right) & 1\end{bmatrix} \qquad ...14$$

Applying the transformation matrix to equation 14 in the same manner as in that of the stator, the equation will be obtained as: (V. Sarac et al 2013)

$$V_r^{qdo}=\left(w-w_r\right)\begin{bmatrix}0&1&0\\-1&0&0\\0&0&0\end{bmatrix}\lambda_r^{qdo}+p\,\lambda_r^{qdo}+r_r^{qdo}\,i_r^{qdo} \qquad ...15a\cdot$$

From equation 15a, the park equations are:
$$\left.\begin{aligned} V_{qr} &= (w-w_r)\lambda_{dr} + p\lambda_{qr} + r_{qr}\, i_{qr} \\ V_{dr} &= -(w-w_r)\lambda_{qr} + p\,\lambda_{dr} + r_{dr}\, i_{dr} \\ V_{or} &= p\,\lambda_{or} + r_{or}\, i_{or} \end{aligned}\right\} \quad ....15b$$

2.5 d-q-o Flux Linkage Relation

The stator qdo flux linkages are obtained by applying $T_{qdo}(\Theta)$ to the stator abc flux linkages of equation (16), that is:

$$\lambda_S^{abc} = L_{SS}^{abc}\; i_S^{abc} + L_{Sr}^{abc}\; i_r^{abc} \quad ...16$$

Using appropriate transformation, we obtain:

$$T(\theta)\,\lambda_S^{abc} = T_{qdo}(\theta) L_{SS}^{abc}\, T(\theta)^{-1}\, i_s^{qdo} + T(\theta) L_{SS}^{abc}\, T(\theta-\theta_r)^{-1}\, i_r^{qdo} \quad ...17$$

$$\lambda_S^{qdo} = [T_{qdo}(\theta)] L_{SS}^{abc} [T_{qdo}(\theta)]^{-1}\, i_S^{qdo} + [T_{qd0}\;(\theta)] L_{Sr}^{abc} [T_{qds}(\theta-\theta_r)]^{-1}\, i_r^{qdo} \quad ...17a$$

Equation (17) gives rise to:

$$\lambda_s^{qdo} = \begin{bmatrix} L_{LS} + \frac{3}{2}L_{SS} & 0 & 0 \\ 0 & L_{SL} + \frac{3}{2}L_{SS} & 0 \\ 0 & 0 & L_{LS} \end{bmatrix} i_S^{qdo} + \begin{bmatrix} \frac{3}{2}L_{Sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{Sr} & 0 \\ 0 & 0 & 0 \end{bmatrix} i_r^{qdo} \quad ...17b$$

Similarly, the qdo rotor flux linkage can be derived as,

$$\lambda_r^{abc} = L_{rS}^{abc}\; i_S^{abc} + L_{rr}^{abc}\, i_r^{abc}.$$

Applying the transformation:

$$T(\theta-\theta_r)\lambda_r^{abc} = T(\theta-\theta_r) L_{sr}\, T(\theta)^{-1}\, i_S^{qdo} + T(\theta-\theta_r) L_{rr}^{abc} T(\theta-\theta_r)^{-1}\, i^{qdo}$$

$$\therefore \lambda_r^{qdo} = [T_{qdo}(\theta-\theta_r)]\, L_{rS}^{abc} [T_{qdo}(\theta)]^{-1}\, i_s^{qdo} + [T_{qdo}(\theta-\theta_r)]^{-1}\, i_r^{qdo} \quad ...18a$$

This gives:

$$\lambda_r^{qdo} = \begin{bmatrix} \frac{3}{2}L_{sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix} i_s^{qdo} + \begin{bmatrix} L_{Lr} + \frac{3}{2}L_{rr} & 0 & 0 \\ 0 & L_{Lr} + \frac{3}{2}L_{rr} & 0 \\ 0 & 0 & L_{Lr} \end{bmatrix} i_r^{qdo} \quad ...18b$$

The stator and rotor flux linkage relationships in equations (17b and 18b) an be expressed compactly as

$$\begin{bmatrix} \lambda_{qS} \\ \lambda_{dS} \\ \lambda_{oS} \\ \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{or} \end{bmatrix} = \begin{bmatrix} L_{LS}+L_m & 0 & 0 & L_m & 0 & 0 \\ 0 & L_{LS}+L_m & 0 & 0 & L_m & 0 \\ 0 & 0 & L_{Ls} & 0 & 0 & 0 \\ L_m & 0 & 0 & L'_{Lr} & 0 & 0 \\ 0 & L_m & 0 & 0 & L'_{Lr}+L_m & 0 \\ 0 & 0 & 0 & 0 & 0 & L'_{Lr} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \\ i'_{qr} \\ i'_{dr} \\ i'_{or} \end{bmatrix} \quad ...19$$

Where,

$$\lambda'_{qr} = \frac{Ns}{Nr}\lambda_{qr}, \quad \lambda'_{dr} = \frac{Ns}{Nr}\;\lambda_{dr} \quad ...20$$

$$i'_{qr} = \frac{Nr}{Ns} i_{qr}, \quad i'_{dr} = \frac{Nr}{Ns}\; i_{dr} \quad ...21$$

$$L'_{Lr} = \left(\frac{Ns}{Nr}\right)^2\; L_{Lr} \quad ...22$$

And L_m, the magnetizing inductance on the stator side, is: $Lm = \frac{3}{2} Lss = \frac{3}{2} \frac{N_s}{N_r} L_{sr} = \frac{3}{2} \frac{N_s}{N_r} L_{rr}$ ...23

From equation (11) and equation (15b),
For stator,

$$V_{qs} = r_s i_{qs} + w\lambda d_s + p\lambda_{qs}$$
$$V_{ds} = r_s i_{qs} + w\lambda_{qs} + p\lambda_{ds}$$
$$V_{os} = r_s i_{os} p\lambda_{os}$$

For rotor

$$V'_{qr} = r'_r i'_{qr} + (w - w_r)\lambda'_{dr} + p\lambda'_{qr}$$
$$V'_{dr} = r'_r i'_{dr} - (w - w_r)\lambda'_{qr} + p\lambda'_{dr}$$
$$V'_{or} = r'_r i'_{or} + p\lambda'_{or}$$

Also, flux linkage of equation 19 can be written as:

$$\left.\begin{aligned} \lambda_{qs} &= (L_{ls} + L_m) i_{qs} + L_m i_{qr} \\ &= L_{ls} i_{qs} + L_m (i_{qs} + i_{qr}) \\ \lambda_{ds} &= (L_{ls} + L_m) i_{ds} + L_m i_{dr} \\ &= L_{ls} i_{ds} + L_m \left(i_{ds} + i'_{dr} \right) \\ \lambda'_{qr} &= L'_{lr} i'_{qr} + L_m (i_{qs} + i'_{qr}) \\ \lambda'_{dr} &= L_{lr} i'_{dr} + L_m (i_{ds} + i_{dr}) \\ \lambda'_{or} &= L_{lr} i'_{or} \end{aligned}\right\} \quad ...19b$$

Substituting the values of flux in the rotor and stator voltage equations (11) and (15b) gives:

$$\left.\begin{aligned} V_{qs} &= r_s i_{qs} + w\lambda_{ds} + p\left(L_{ls} i_{qs} + L_m (i_{qs} + l'_{qr}\right) \\ V_{qr}' &= r'_r i'_{qr} + (w - w_r)\lambda'_{dr} + p\left[L_{lr} i_{qr} + L_m \left(i_{qs} + i'_{qr}\right)\right. \\ V_{ds} &= r_s i_{ds} - w\lambda_{qs} + p\left[(L_{ls} + L_m) i_{ds} + L_m i_{dr}\right] \\ V'_{dr} &= r'_r i'_{dr} - (w - w_r)\lambda'_{qr} + p\left[L_{lr} i'_{dr} + L_m \left(i_{ds} + i'_{dr}\right)\right. \\ V_{os} &= r_s i_{os} + p(L_s i_{os}) \\ V'_{or} &= r'_r i'_{or} + p(L_{lr} i'_{or}) \end{aligned}\right\} \quad ...24$$

In terms of x's and ψ's stator and rotor voltage equations (.24) can be written as:

$$\left.\begin{aligned} v_{qs} &= \frac{p}{w_b}\psi_{ds} + \frac{w}{w_b}\psi_{ds} + r_s i_{qs} \\ v_{ds} &= \frac{p}{w_b}\psi_{ds} - \frac{w}{w_b}\psi_{qs} + r_s i_{ds} \\ v_{0s} &= \frac{p}{w_b}\psi_{0s} + r_s i_{0s} \\ v'_{qr} &= \frac{p}{w_b}\psi'_{qr} + \left(\frac{w - w_r}{w_b}\right)\psi'_{dr} + r'_r i'_{qr} \\ v'_{dr} &= \frac{p}{w_b}\psi'_{dr} - \left(\frac{w - w_b}{w_b}\right)\psi'_{qr} + r'_r i'_{dr} \\ v'_{0r} &= \frac{p}{w_b}\psi'_{0r} + r'_r i'_{0r} \end{aligned}\right\} \quad ...25$$

Where

$$\begin{bmatrix}\psi_{qs}\\ \psi_{ds}\\ \psi_{os}\\ \psi'_{qr}\\ \psi'_{dr}\\ \psi'_{or}\end{bmatrix} = \begin{bmatrix} X_{ls}+X_m & 0 & 0 & X_m & 0 & 0\\ 0 & X_{ls}+X_m & 0 & 0 & X_m & 0\\ 0 & 0 & X_{ls} & 0 & 0 & 0\\ X_m & 0 & 0 & X'_{Lr}+X_m & 0 & 0\\ 0 & X_m & 0 & 0 & X'_{Lr}+X_m & 0\\ 0 & 0 & 0 & 0 & 0 & X'_{Lr}\end{bmatrix} \begin{bmatrix} i_{qs}\\ i_{ds}\\ i_{0s}\\ i'_{qr}\\ i'_{dr}\\ i'_{or}\end{bmatrix} \quad ..26$$

Where $\psi = w_b \lambda$

2.6–q-d-0 Torque Equation in Arbitrary Reference Frame

The sum of the instantaneous input power to all six windings of the stator and rotor is given by

$$P_{in} = V_{as}\, i_{as} + V_{bs}\, i_{bs} + V_{cs}\, i_{cs} + V'_{ar}\, i'_{ar} + V'_{c}\, i'_{cr} \quad ...27$$

In terms of q-d-0 quantities, the instantaneous input power is

$$Pin = \frac{3}{2}\left(V_{qs}\, i_{qs} + V_{ds}\, i_{ds} + 2V_{os}\, i_{os} + V'_{qr}\, i'_{qr} + V'_{dr}\, i'_{dr} + 2V'_{or}\, i_{or}\right) ...28$$

Substituting the values of $V_{qs}, V_{ds}, V'_{qr}, V'_{dr}$ from equation (10) and (15a) into the right side of equation (28), we obtain;

$$P_{in} = \frac{3}{2}\left(i_{qs}\left(r_s\, i_{qs} + w\lambda_{ds} + p\lambda_{qs}\right) + i_{ds}\left(r_s\, i_{ds} - w\lambda_{qs} + p\lambda_{ds}\right) + i'_{dr}\right.$$
$$\left.\left(r_r\, i'_{qr} + r_r\, i'_{qr} + \left(w - w_r\right)\lambda'_{dr} + \left(r_r\, i'_{dr} - \left(w - w_r\right)\lambda'_{qr} + p\lambda'_{dr}\right)\right]\right. \quad ...29$$

From equation (29), we obtain 3 kinds of terms: i^2r, i $p\ \lambda$, and $\omega\ \lambda$ i terms. The i^2r terms are copper losses; the i $p\ \lambda$ terms represent the rate of exchange of magnetic field energy between windings. The w λ i terms represent the rate of energy converted to mechanical work. That is

$$T_{em} = \frac{3}{2}.\frac{1}{w_m}\left[w\left(\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds}\right) + \left(w - w_r\right)\left(\lambda'_{dr}\, i_{qr} - \lambda'_{qr}\, i'_{dr}\right)\right]$$

But $w_m = \dfrac{2w_b}{p}$

$$\therefore Tem = \frac{3}{2}\frac{P}{2wb}\left[w\left(\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds}\right) + \left(w - w_r\right)\left(\lambda'_{dr}\, i_{qr} - \lambda_{qr}\, i_{dr}\right)\right] \quad ...30$$

Substituting values of $\lambda_{qs}, \lambda_{ds}, \lambda'_{qr}, \lambda'_{dr}$ from equation (19b) into equation 30, we have

$$\left[\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds}\right] = L_{ls}\, i_{ds}\, i_{qs} + L_m \left[i_{ds}\, i_{qs}\, i'_{dr}\, i_{qs}\right]$$
$$- \left[L_{ls}\, L_{qs}\, i_{ds} + L_m\, i_{qs}\, i_{ds} + L_m\, i_{qs}\right]$$
$$= L_m \left[i_{qs}\, i_{ds} - i_{ds}\, i'_{qr}\right]$$

And

$$\lambda'_{qr}\, i_{dr} - \lambda'_{dr}\, i_{qr} = L_m \left(i_{qs}\, i_{dr} - i_{ds}\, i_{qs}\right)$$

This implies that

$$\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds} = -\left[\lambda'_{dr}\, i_{qr} - \lambda'_{qr}\, i_{ds}\right]$$
$$\lambda'_{qr}\, i_{dr} - \lambda'_{dr}\, i_{qr} = L_m \left(i_{qs}\, i'_{dr} - i_{dr}\, i'_{dr}\right]$$

Therefore,

$$\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds} = -\left(\lambda'_{dr}\, i'_{qr} - \lambda'_{qr}\, i'_{dr}\right) = L_m \left(i_{dr}\, i_{qr} - i_{qr}\, i_{ds}\right) \quad ...31$$

Substituting equation (31) into equation (30) gives;

$$\left.\begin{aligned} Tem &= \frac{3}{2}\frac{P}{2}\left(\lambda'_{qr}\, i'_{dr} - \lambda'_{qr}\, i_{qr}\right) N.m \\ &= \frac{3}{2}\frac{P}{2}\left(\lambda_{ds}\, i_{qs} - \lambda_{qs}\, i_{ds}\right) N.m \\ &= \frac{3}{2}\frac{P}{2} L_m \left(i'_{dr}\, i_{qs} - i'_{qr}\, i_{ds}\right) N.m \end{aligned}\right\} \quad ...32$$

3–Induction Machine Equation in Stationary Reference Frame

By setting w=0 in equation (21) gives:

$$\left.\begin{aligned} V^s_{qs} &= r_s\, i_{qs} + p\lambda_{qs} \\ V^s_{ds} &= r_s\, i_{ds} + p\lambda_{ds} \\ V^s_{os} &= r_s\, i_{os} + p\lambda_{os} \end{aligned}\right\} \quad ...33$$

$$V^s_{qs} = r_s\, i_{qs} + p\frac{w_b}{w_b}\lambda_{qs}$$

$$\Rightarrow \; V^s_{qs} = r_s\, i_{qs} + \frac{p}{w_b}\psi_{qs}$$

Where $\psi = w_b \lambda$

Therefore, for stator

$$\left.\begin{aligned} V^s_{qs} &= \frac{p}{w_b}\psi_{qs} + r_s\, i'_{qs} \\ V^s_{ds} & \frac{p}{w_b}\psi_s + r_s\, i^s_{ds} \\ V_{os} &= \frac{p}{w_b}\psi_{os} + r_s\, i_{os} \end{aligned}\right\} \quad ...34$$

For rotor

Set w=0 in equation (15b):

$$\left.\begin{aligned} V_{qr}^{'s} &= \frac{p}{w_b}\psi_{qr}^{s} - \frac{w_r}{w_b}\psi_{dr}^{'s} + r_r^{'}\, i_{dr}^{'s} \\ V_{dr}^{'s} &= \frac{p}{w_b}\psi_{qr}^{'s} + \frac{w_r}{w_b}\psi_{qr}^{'s} + r_r^{'}\, i_{dr}^{'s} \\ V_{or} &= \frac{p}{w_b}\psi_{or} + r_r^{'}\, i_{or}^{'} \end{aligned}\right\} \qquad ...35$$

$$\begin{bmatrix} \psi_{qs}^{s} \\ \psi_{ds}^{s} \\ \psi_{os}^{s} \\ \psi_{qr}^{'s} \\ \psi_{dr}^{'s} \\ \psi_{or}^{'s} \end{bmatrix} = \begin{bmatrix} x_{ls}+x_m & 0 & 0 & x_m & 0 & 0 \\ 0 & x_{ls}+x_m & 0 & 0 & x_m & 0 \\ 0 & 0 & x_{ls} & 0 & 0 & 0 \\ x_m & 0 & 0 & x_{Lr}^{'}+x_m & 0 & 0 \\ 0 & x_m & 0 & 0 & x_{Lr}^{'}+x_m & 0 \\ 0 & 0 & 0 & 0 & 0 & x_{Lr}^{'} \end{bmatrix} \begin{bmatrix} i_{qs}^{s} \\ i_{ds}^{s} \\ i_{0s}^{s} \\ i_{qr}^{'s} \\ i_{dr}^{'s} \\ i_{or}^{'} \end{bmatrix} \qquad ...36$$

Torque equations,

$$\begin{aligned} Tem &= \frac{3}{2}\frac{P}{2w_b}\left(\psi_{dr}^{'s} - \psi_{dr}^{'s}\, i_{qr}^{'s}\right) \\ &= \frac{3}{2}\frac{P}{2w_b}\left(\psi_{ds}^{s}\, i_{qs}^{s} - \psi_{qs}\, i_{ds}^{s}\right) \\ &= \frac{3}{2}\frac{P}{2w_b}\, x_m \left(i_{dr}^{'s}\, i_{qs} - i_{qr}^{'s}\, i_{ds}^{s}\right) N.m \end{aligned} \qquad ...37$$

3.1 – Equations for Simulation of Induction Motor in Stationary Reference Frame

The model equations of the induction machine in the stationary **q-d-0** reference frame may be rearranged into the following form for simulation

$$\left.\begin{aligned}
\Psi_{qs}^{s} &= w_b \int (V_{qs}^{s} + \frac{r_s}{x_{LS}} (\Psi_{mq}^{s} - \Psi_{qs}^{s}))\, dt \\
\Psi_{ds}^{s} &= w_b \int (V_{ds}^{s} + \frac{r_s}{x_{LS}} (\Psi_{md}^{s} - \Psi_{ds}^{s}))\, dt \\
i_{os} &= \frac{w_b}{x_{LS}} \int (V_{os} - l_{os}\, r_s)\, dt
\end{aligned}\right] \quad ...38$$

$$\left.\begin{aligned}
\Psi_{qr}^{s} &= w_b \int (V_{qr}^{'s} + \frac{w_r}{w_b} \Psi_{dr}^{'s} + \frac{r_r^{'}}{x_{Lr}^{'}} (\Psi_{mq}^{s} - \Psi_{qr}^{s}))\, dt \\
\Psi_{dr}^{'s} &= w_b \int (V_{ds}^{'s} + \frac{w_r}{w_b} \Psi_{dr}^{'s} + \frac{r_r^{'}}{x_{Lr}^{'}} (\Psi_{md}^{s} - \Psi_{dr}^{'s}))\, dt \\
i_{or} &= \frac{w_b}{x^{'}_{Lr}} \int (V^{'}_{or} - i_{or}\, r_r^{'})\, dt
\end{aligned}\right] \quad ...39$$

$$\left.\begin{aligned}
\Psi_{mq}^{s} &= x_m\, (i_{qs}^{'s} + i_{qr}^{'s}) \\
\Psi_{mq}^{s} &= x_m\, (i_{ds}^{'s} + i_{dr}^{'s})
\end{aligned}\right] \quad ...40$$

$$\left.\begin{aligned}
\Psi^{s}_{qs} &= x_{Ls}\, i_{qs}^{'} + \psi_{mq}^{s}, & i_{qs}^{'s} &= \frac{\Psi_{qs}^{s} - \psi_{mq}^{s}}{x_{Ls}} \\
\Psi_{ds}^{s} &= x_{Ls}\, i_{ds}^{s} + \psi_{md}^{s}, & i_{ds}^{'s} &= \frac{\Psi_{qs}^{s} - \psi_{md}^{s}}{x_{Ls}} \\
\Psi_{qr}^{'s} &= \left(x_{Lr}^{'}\right) i_{qr}^{s} + \psi_{mq}^{s}, & i_{qr}^{'s} &= \frac{\Psi_{qr}^{'s} - \psi_{mq}^{s}}{(x_{Lr}^{'})} \\
\Psi_{dr}^{'s} &= \left(x_{Lr}^{'}\right) i_{dr}^{'s} + \Psi_{dr}^{,s}, & i_{dr}^{'s} &= \frac{\Psi_{dr}^{'s} - \psi_{md}^{s}}{(x_{Lr}^{'})}
\end{aligned}\right] \quad ...41$$

Where

$$\frac{1}{x_M} = \frac{1}{x_m} + \frac{1}{x_{ls}} + \frac{1}{x_{lr}^{'}} \quad ...42$$

and

$$\psi_{mq}^{s} = x_M \left(\frac{\psi_{qs}^{s}}{x_{ls}} + \frac{\psi_{dr}^{'s}}{x_{lr}^{'}} \right)$$

$$\psi_{md}^{s} = x_M \left(\frac{\psi_{ds}^{s}}{x_{ls}} + \frac{\psi_{dr}^{'s}}{x_{lr}^{'}} \right) \quad ...43$$

The torque equation is

$$T_{em} = \frac{3}{2}\ \frac{P}{2w_b}\ (\Psi_{qs}^{s}\ i_{ds}^{s} - \Psi_{qs}^{s}\ i_{ds}^{s})\quad N.m$$

The equation of motion of the rotor is obtained by equating the inertia torque to the accelerating torque, that is,

$$\frac{dw_{rm}}{dt} = T_{em} + T_{mech} - T_{damp} \qquad N.m \qquad ...44$$

Where T_{mech} = the externally applied mechanical torque in the direction of the rotor speed.
T_{damp} = the damping torque in the direction opposite to rotation.

When equation (44) is used in conjunction with equation (38) and (39), the per unit speed, w_r/w_b needed for building the speed voltage terms in the rotor voltage equations, can be obtained by integrating

$$\frac{2Jw_b}{P}\frac{d\left(w_r / w_b\right)}{dt} = Tem + Tmech - Tdamp \quad N.m \qquad ...45$$

Equation (45) can be written in terms of the inertia constant, H, defined as the ratio of the kinetic energy of the rotating mass at base speed to the rated power, that is

$$H = \frac{Jw_{bm}^2}{2S_b} \quad in \ \ per.unit \qquad ...46$$

Expressed in per unit values of the machine's own base power and voltage, equation (45) can be rewritten as

$$2H\,\frac{d\left(w_r / w_b\right)}{dt} = T_{em} + T_{mech} - T_{damp} \quad in\ per\ unit \quad ...47$$

4 – SIMULATION RESULTS

The reference model we discussed exhaustively above, was tested on a three phase wound rotor (slip-ring) induction motor, with rating tabulated in table 1 below, to analyze the dynamic behavior of the machine.

Table 1 – Circuit Parameter Values and constants for the dynamic simulation of the three-phase wound rotor (slip-ring) induction motor (Eleanya 2015),

Circuit parameter	Value
Power rating	5hp
Voltage (Line to line) (V)	415v
Frequency (f)	50Hz
No of poles (P_n)	4
Stator resistance (r_s)	0.22Ω
Stator inducdance (L_s)	0.0425H
Rotor resistnace r_i	0.209Ω
Rotor induction Lŕ	0.0430H
Magnetizing inductance Lm	0.040H
Moment of inertia of motor J	0.124kgm^2

With values of table 1 and using equation 38 through equations 47, the dynamic simulation plots for the machine is as shown in fig 1 through fig 4

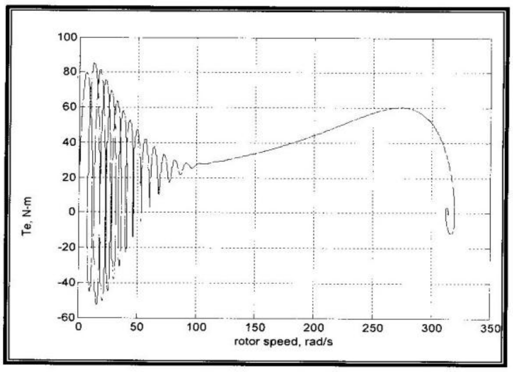


Fig 1: The Electromagnetic Torgue, T_e against rotor speed for the three-phase wound rotor

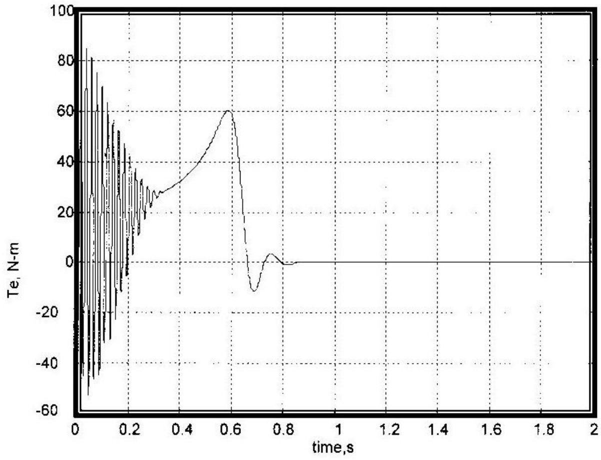


Fig 2: The Electromagnetic Torgue, T_e against time for the three-phase wound rotor Induction

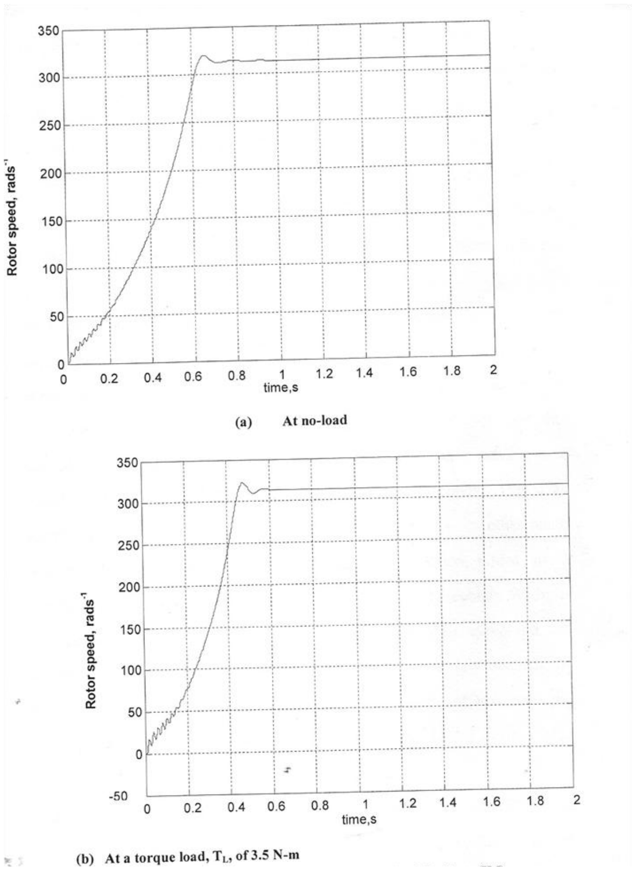
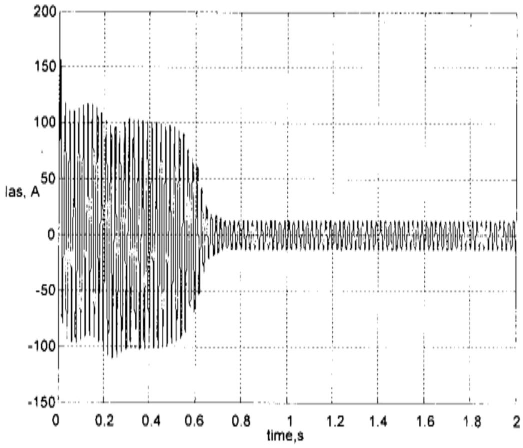
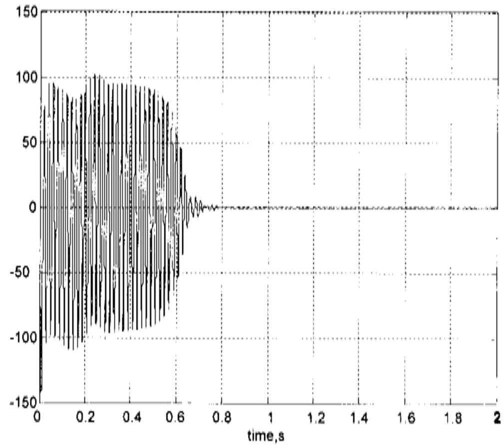


Fig 3: Rotor against time for three-phase wound rotor induction Motor



(a) Phase A Stator Current, I_{as}



(b) Phase A rotor current, I_{ar}

Fig 4: Plotting of phase A currents for stator and rotor windings of the induction motor

5 – PERFORMANCE ANALYSIS OF THE MACHINE UNDER TRANSIENT CONDITION

From the graphs plotted, and irrespective of the reference frame chosen for the motor, the material inductance of machine depends on the rotor position which varies with time.

Due to transient condition, the torque Vs speed plotting exhibited characteristics similar to that obtainable under steady-state condition.

The machine oscillated and settled at a rotor speed of 314 rad s^{-1} That is at synchronous speed. The plotting is shown in fig 1.

When the machine was simulated to start on no load, it accelerated freely, and after some damped oscillation, settles at a synchronous speed, $w_r = w_s = 314$ rad s^{-1} at a time of 0.6 seconds. See fig 3a On the other round, when it was simulated with load torque of 3.5Nm, the rotor speed did not accelerate freely as it did under no-load; rather it has few seconds of delayed acceleration, oscillated very briefly and settles at less than the synchronous speed ($w_r = 310$ rad s^{-1}), at oscillation time of 1.2 seconds. This is shown in fig 3b.

More-still, the simulation study also unveils that the machine cannot actually tolerate a longer starting time with out undue disturbance to the supply, due to its high starting current characteristic, especially under blocked rotor condition. The plotting is illustrated in fig 4a/b.

On no-load and final operating speed, the current circulating in the rotor winding of the machine is zero.

Further still, the electromagnetic torque of the machine, against time is moderately high at start, and assumed a steady condition after 0.8 seconds. This is illustrated in fig 2.

6 – Conclusion

This work has presented vividly a simple method of analyzing and simulating the transient state performance of three-phase wound rotor (slip-ring) induction motor under operating condition. The simulation results presented in this report are indeed indispensable and predictable tools to Electrical Machine Engineers over the motor performance prior to their designs and constructions.

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