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Determination of Drop Size fromBatch Settling Velocity

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Abstract: Two correlations for the prediction of Sauter mean diameter in liquid-liquid dispersion have been proposed in terms of physical properties, batch sedimentation velocity, and hold–up of dispersed phase. These correlations were verified with 127 data points for nine liquid–liquid systems from four different sources and gave the lowest deviation in mean drop sizes compared with other correlations from literature.

Keywords: Drop size, Settling velocity, liquid-liquid dispersion, Sauter mean diameter, Extraction. Hold–up

1. Introduction

A variety of drop size measurement techniques are available which had been used in academic work [1]. In studies involving processes such as uranium, copper, nickel and cobalt recovery by solvent extraction in mixer settlers, the two difficulties to be overcome in drop size measurements are the presence of dark – colored liquids (usually in the continuous phase) and dispersed phase hold–up near 50% [2]. Two methods appeared in literature to overcome this problem based on batch settling test. In batch settling test, once a liquid – liquid system has been agitated for the required period, the agitator is turned off, and the drops of the dispersed phase travels away from the sedimentation interface and towards the coalescence interface where the droplets coalesce with the homophase. This hindered settling aspect was not considered as relevant since it was assumed that, the vertical motion of the droplets is controlled by the rate of coalescence on the coalescence front. This hindered settling velocity depends on diameter of drops, hold–up, and physical properties of systems investigated [3]. The first study was performed by Grossman[4], who determined the drop size distribution from batch separation test for a dispersion of Feron 113 in water (ε_o : 3 – 5%). His analysis is based on the following assumptions ; the settling velocity of droplets is a function of its size, the properties of both liquids are fixed with time, the initial acceleration of the droplets is neglected and there are no collisionsor interactions between droplets. The last assumption is good when the hold-up of the dispersed phase is small and the variation in size between the droplets are no too large. By recording the height of the sedimentation interface with time, assuming a relation between drop size and sedimentation according to Stoke's law corrected by Hadamard [5] as :

$$u_{sh} = k_h r^2 \qquad (1)$$

$$k_h = \frac{2}{9}\left(\frac{g}{\mu_c}\right)\left(\frac{(\rho_c - \rho_d)(1 + \mu_d/\mu_c)}{(0.66 + \mu_d/\mu_c)}\right) \qquad (2)$$

and assuming a certain drop size distribution according to Dallavalle et al. [6];

$$N^*(r) = N^*{}_o\left(\frac{r}{r_o}\right)^n \left(\exp\left[-(n/a)\left(\frac{r}{r_o}\right)^a\right]\right. \qquad (3)$$

Grossman [4] was able to formulate the following theoretical equation for sedimentation boundary :

$$x / x_o = \frac{(1 - \psi/\varepsilon)}{(1 - \psi)} \qquad (4)$$

$$\psi = \varepsilon * 0.75\pi r^2 N^*(r) \frac{(1 - r^2)}{r^{*2}} \delta r \qquad (5)$$

$$r^* = \left[\frac{(x_o - x)}{k_h t}\right]^{0.5} \qquad (6)$$

The method of analysis was based on plotting the sedimentation curve ($x / x_o\varepsilon$) against (t / t_o) for experimental data and then applying equation (4) after assuming a set of parameters, namely characteristics droplet radius r_o, relative variance Γ, and symmetry coefficient Ω. The latter two terms are defined by:

$$\Gamma = 1[1 + \frac{(\omega - 1)}{2\xi}(1 - (1 + \frac{4\omega\xi}{(\omega - 1)^2})^{1/2})]^{1/\omega} \qquad (7)$$

$$\Omega = [\frac{[1 + [(\omega - 1)/2\xi](1 + (4\omega\xi/(\omega - 1)^{1/2}))]^{1/\omega} - 1}{1 - [1 + ((\omega - 1)/2\xi)(1 - (1 + (4\xi\omega/(\omega - 1)^2)^{1/2}]^{1/\omega}} \qquad (8)$$

Each experimental curve was compared with large number of such theoretical curves with different sets of parameters, and the parameters corresponding to the one with least deviation was characteristic of the drop size distribution of that experiment.Size distribution determined by the author from photography showed similar values to those calculated from equation (4).

Assenov and slater [2] performed another study for droplet size determination at higher values of hold- up (ε : 0.47 – 0.66) in mixer part of mixer–settler extractor. The method is based on assuming a drop size distribution equation taking the following form [7] :

$$f\left(\frac{\phi}{\phi_m}\right) = \left(\frac{1}{6\beta^4}\right)\left(\frac{\phi}{\phi_m}\right)^3 \exp\left(-\frac{\phi}{\beta\phi_m}\right) \qquad (9)$$

Where β was assumed to take a value of 0.13, and $\phi_{32} = 3\beta\phi_m$. By specifying values for ϕ_{32}, the distribution function was calculated. The average terminal velocity for each size increment in the proposed distribution was determined according to its Reynold's number. For each terminal velocity a sedimentation velocity was predicted from Richardson and Zeki[8] equation for sedimentation of solid particles in liquid media:

$$u_{bs} = u_s (1-\varepsilon_s)^q \tag{10}$$

An average batch sedimentation velocity could then be expressed as a function of mean drop size. From batch experiment, the batch sedimentation velocity was determined in the region of constant velocity. This is occurred after short time of initiation of batch experiment when the effect of turbulence disappeared. A trial and error procedure was then carried out and the mean drop size, which give a sedimentation velocity equal to that of experiment, represent the mean drop size of dispersion. The authors found a reasonable agreement between drop sizes determined from above procedure and those from photographs.

2. Dataand analysis

An attempt was made to use the correlations for prediction of slip velocity for the evaluation of mean drop size in different liquid – liquid dispersions. Table (1) reviewed such correlations from different sources. The correlation of Kim and Choi [13] was based on data for slip velocities in batch dispersion decay in the mixer part of mixer-settler extractor. While all other correlation listed in Table (1) was based on spray column data. In spray columns, the drop sizes, and hence Reynolds number are relatively higher than those in mixer–settler extractors.

The drag coefficient of multi–drop system can be related to Reynolds number by the following equation:

$$C_D = \upsilon \mathrm{Re}_\phi^{\lambda} \tag{11}$$

Where Re_ϕ and C_D were given by Barnea and Mizrahi [9] and Kumar [11] (Table 1).

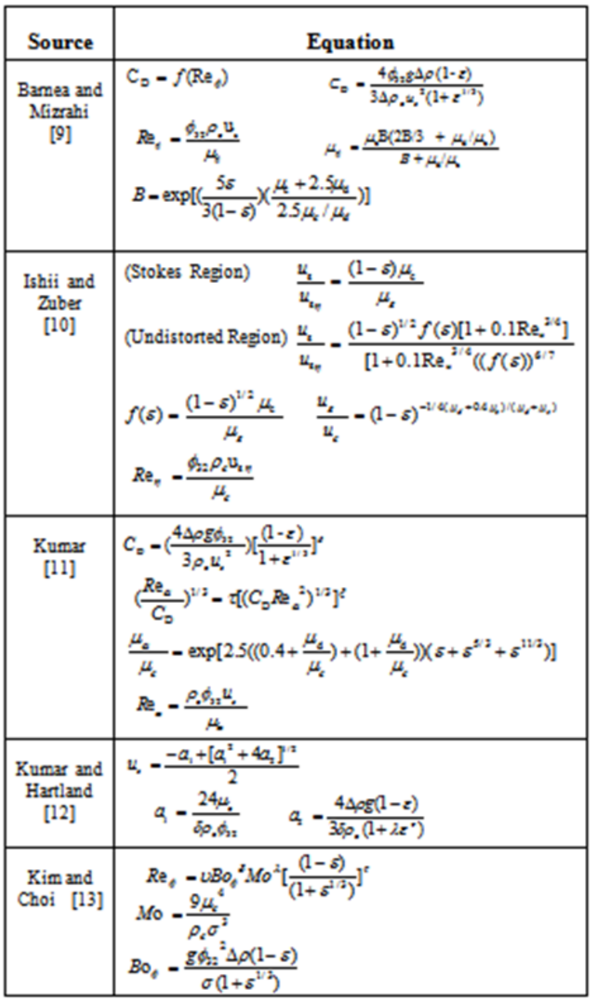
Equation (11) can be modified by introducing Bond number, as a measure of interfacial tension forces. The resulting equation will be:

$$C_D = \upsilon \mathrm{Re}_\phi^{\lambda} \mathrm{Bo}^{\tau} \tag{12}$$

Where:

$$Bo = \frac{g\phi_{32}\Delta\rho}{\sigma} \tag{13}$$

Table (1): Some published correlations for prediction of slip velocity in liquid-liquid dispersions.



Source	Equation
Barnea and Mizrahi [9]	$C_D = f(Re_\phi)$ $\quad C_D = \frac{4\phi_{32} g\Delta\rho(1-\varepsilon)}{3\Delta\rho_c u_s^2(1+\varepsilon^{1/3})}$ $Re_\phi = \frac{\phi_{32}\rho_c u_s}{\mu_\phi}$ $\quad \mu_\phi = \frac{\mu_c B(2B/3 + \mu_d/\mu_c)}{B+\mu_d/\mu_c}$ $B = \exp[(\frac{5\varepsilon}{3(1-\varepsilon)})(\frac{\mu_d+2.5\mu_c}{2.5\mu_c/\mu_d})]$
Ishii and Zuber [10]	(Stokes Region) $\frac{u_s}{u_{t\infty}} = \frac{(1-\varepsilon)\mu_c}{\mu_m}$ (Undistorted Region) $\frac{u_s}{u_{t\infty}} = \frac{(1-\varepsilon)^{1/2} f(\varepsilon)[1+0.1Re_\infty^{3/4}]}{[1+0.1Re_\infty^{3/4}((f(\varepsilon))^{6/7}}$ $f(\varepsilon) = \frac{(1-\varepsilon)^{1/2}\mu_c}{\mu_m}$ $\quad \frac{\mu_m}{\mu_c} = (1-\varepsilon)^{-1/4(\mu_d+0.4\mu_c)/(\mu_d+\mu_c)}$ $Re_\infty = \frac{\phi_{32}\rho_c u_{t\infty}}{\mu_c}$
Kumar [11]	$C_D = (\frac{4\Delta\rho g\phi_{32}}{3\rho_c u_s^2})[\frac{(1-\varepsilon)}{1+\varepsilon^{1/3}}]^\tau$ $(\frac{Re_\phi}{C_D})^{1/3} = \tau[(C_D Re_\phi^2)^{1/3}]^\xi$ $\frac{\mu_\phi}{\mu_c} = \exp[2.5((0.4+\frac{\mu_d}{\mu_c})+(1+\frac{\mu_d}{\mu_c}))(\varepsilon+\varepsilon^{5/3}+\varepsilon^{11/3})]$ $Re_\phi = \frac{\rho_c\phi_{32}u_s}{\mu_\phi}$
Kumar and Hartland [12]	$u_s = \frac{-a_1+[a_1^2+4a_2]^{1/2}}{2}$ $a_1 = \frac{24\mu_c}{\delta\rho_c\phi_{32}}$ $\quad a_2 = \frac{4\Delta\rho g(1-\varepsilon)}{3\delta\rho_c(1+\lambda\varepsilon^\kappa)}$
Kim and Choi [13]	$Re_\phi = \upsilon Bo_\phi^\lambda Mo^\lambda[\frac{(1-\varepsilon)}{(1+\varepsilon^{1/3})}]^\tau$ $Mo = \frac{9\mu_c^4}{\rho_c\sigma^3}$ $Bo_\phi = \frac{g\phi_{32}^2\Delta\rho(1-\varepsilon)}{\sigma(1+\varepsilon^{1/3})}$

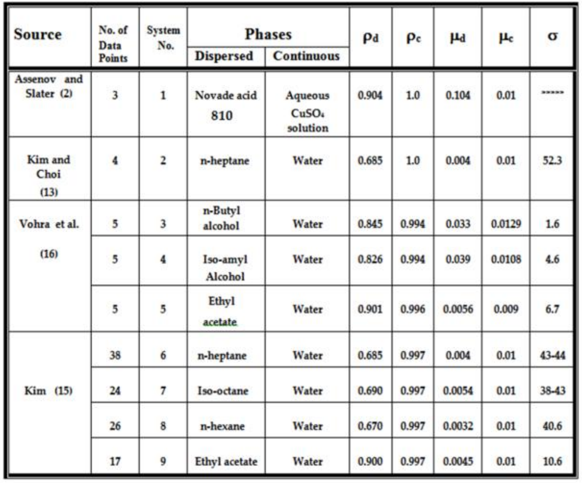
127 experimental data points were collected from literature which includes the measurements of Sauter mean diameter and the rate of movement of sedimentation boundary (u_{bs}) in batch dispersion. These data were collected from four different sources which includes nine liquid-liquid systems.The physical properties and the operating range of systems investigated are shown in Table (2)and (3) respectively. For these data, the slip velocity (u_s) can be obtained from the following equation [14, 15]:

$$u_s = \frac{u_{bs}}{(1-\varepsilon_s)} \tag{14}$$

Where$\varepsilon_s$represent the hold–up in the sedimentation zone of a batch decay where the velocity is approximately constant (the first region of sedimentation). The average hold – up in this region (ε_s) was assumed to be equal to the initial hold-up of dispersion [2].

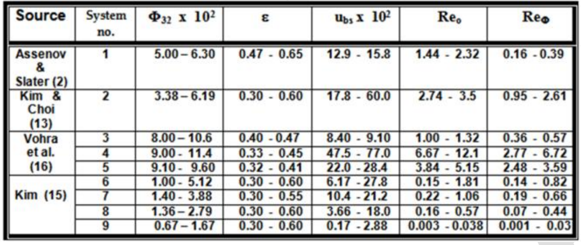
New coefficients and indices were determined for the correlations of Kumar [11] and Kumar and Hartland [12] by trialand error procedure using

Table (2): Systems investigated ; Physical properies (cgs units).



Source	No. of Data Points	System No.	Phases		ρ_d	ρ_c	μ_d	μ_c	σ
			Dispersed	Continuous					
Assenov and Slater (2)	3	1	Novade acid 810	Aqueous $CuSO_4$ solution	0.904	1.0	0.104	0.01	-----
Kim and Choi (13)	4	2	n-heptane	Water	0.685	1.0	0.004	0.01	52.3
Vohra et al. (16)	5	3	n-Butyl alcohol	Water	0.845	0.994	0.033	0.0129	1.6
	5	4	Iso-amyl Alcohol	Water	0.826	0.994	0.039	0.0108	4.6
	5	5	Ethyl acetate	Water	0.901	0.996	0.0056	0.009	6.7
Kim (15)	38	6	n-heptane	Water	0.685	0.997	0.004	0.01	43-44
	24	7	Iso-octane	Water	0.690	0.997	0.0054	0.01	38-43
	26	8	n-hexane	Water	0.670	0.997	0.0032	0.01	40.6
	17	9	Ethyl acetate	Water	0.900	0.997	0.0045	0.01	10.6

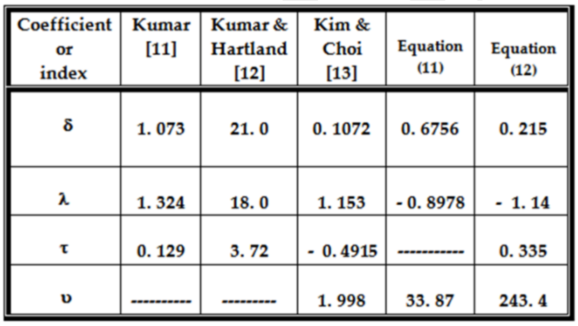
Table (3):Systems investigated : Operating conditions (cgs).



Source	System no.	Φ_{32} x 10^2	ε	u_{bs} x 10^2	Re_o	Re_Φ
Assenov & Slater (2)	1	5.00 – 6.30	0.47 - 0.65	12.9 - 15.8	1.44 - 2.32	0.16 - 0.39
Kim & Choi (13)	2	3.38 – 6.19	0.30 - 0.60	17.8 - 60.0	2.74 - 3.5	0.95 - 2.61
Vohra et al. (16)	3	8.00 – 10.6	0.40 - 0.47	8.40 - 9.10	1.00 - 1.32	0.36 - 0.57
	4	9.00 - 11.4	0.33 - 0.45	47.5 - 77.0	6.67 - 12.1	2.77 - 6.72
	5	9.10 - 9.60	0.32 - 0.41	22.0 - 28.4	3.84 - 5.15	2.48 - 3.59
Kim (15)	6	1.00 - 5.12	0.30 - 0.60	6.17 - 27.8	0.15 - 1.81	0.14 - 0.82
	7	1.40 - 3.88	0.30 - 0.55	10.4 - 21.2	0.22 - 1.06	0.19 - 0.66
	8	1.36 – 2.79	0.30 - 0.60	3.66 - 18.0	0.16 - 0.57	0.07 - 0.44
	9	0.67 – 1.67	0.30 - 0.60	0.17 - 2.88	0.003 - 0.038	0.001 - 0.03

random search method. While the coefficients and indices of the correlation of Kim and Choi [13] and equations (11) and (12) were estimated by multiple linear regression analysis. All these coefficients and indices are listed in Table (4).

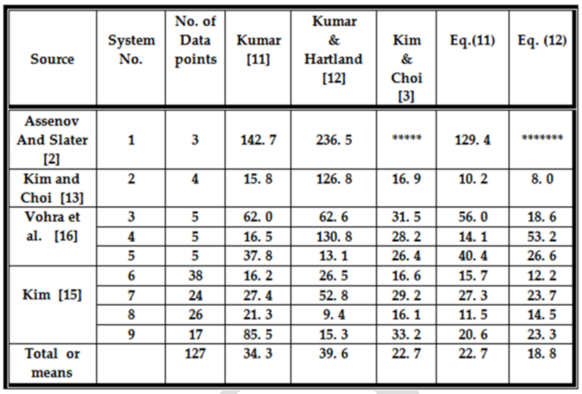
Table (4): Coefficients and Indices obtained for different correlations.



Coefficient or index	Kumar [11]	Kumar & Hartland [12]	Kim & Choi [13]	Equation (11)	Equation (12)
δ	1. 073	21. 0	0. 1072	0. 6756	0. 215
λ	1. 324	18. 0	1. 153	- 0. 8978	- 1. 14
τ	0. 129	3. 72	- 0. 4915	----------	0. 335
υ	----------	---------	1. 998	33. 87	243. 4

The Sauter mean diameters was then estimated from above correlations and compared with experimental values. In Table (5), the accuracy of correlations was compared in terms of average absolute percentage deviation (γ) in the values of Sauter mean drop diameter. Higher discrepancy was observed between actual and predicted values for the data of AssenovandSlater [2]. This maybe attributed to the higher value of dispersed phase viscosity compared with other liquid – liquid systems (Table 2). On comparing Equation (11) and (12) (excluding the data of Assenov and Slater [2]), it seems that introducing interfacial tension forces in Equation (11) did not improve the prediction efficiency significantly (Equation (11), γ = 20.1; Equation (12), γ = 18.7). This implying that, the effect of interfacial tension forces at such low range of Reynolds numbers (0.001< Re_ϕ< 6.72) is not significant. Kumar [11], and Kumar and Hartland [12] also observed such behavior at low and intermediate ranges of Reynolds numbers. For the whole data Equation (11) and (12) give relatively lower deviation in Sauter mean diameter compared with other correlations.

Table (5): Comparison between different correlations



Source	System No.	No. of Data points	Kumar [11]	Kumar & Hartland [12]	Kim & Choi [3]	Eq.(11)	Eq. (12)
Assenov And Slater [2]	1	3	142. 7	236. 5	*****	129. 4	*******
Kim and Choi [13]	2	4	15. 8	126. 8	16. 9	10. 2	8. 0
Vohra et al. [16]	3	5	62. 0	62. 6	31. 5	56. 0	18. 6
	4	5	16. 5	130. 8	28. 2	14. 1	53. 2
	5	5	37. 8	13. 1	26. 4	40. 4	26. 6
Kim [15]	6	38	16. 2	26. 5	16. 6	15. 7	12. 2
	7	24	27. 4	52. 8	29. 2	27. 3	23. 7
	8	26	21. 3	9. 4	16. 1	11. 5	14. 5
	9	17	85. 5	15. 3	33. 2	20. 6	23. 3
Total or means		127	34. 3	39. 6	22. 7	22. 7	18. 8

3. Conclusions

The following conclusions were arrived from this approach:

-Two dimensionless group equations were proposed for evaluation of drop size in terms of settling velocity of drops (Equation (11) and (12)).

- 127 data points collected from literature were used to evaluate the above two equation and the equation of Kumar [11], Kumar and Hartland [12], and Kim and Chio [13] where new coefficients and indices were found for the specified authors equations.
- For the whole data Equations (11) and (12) gives relatively lower deviation.
- On comparing Equation (11) and (12), excluding the data of Assenov and Slater [2], it appeared that introducing the effect of interfacial tension**at**0.001<Re_ϕ< 6.7 is insignificant.

Nomenclature

a,n: parameters defined distribution
Bo : Bond number.
Bo_f : Modified Bond number.
C_D : Drag coefficient.
g : Acceleration due to gravity, cm/sec^2.
k_h: Hadamard correction factor.
Mo : Morton number.
$N^*(r)$: Droplet size distribution function.
N^*_o : Distribution parameter.
q : Exponent.

r^* : Maximum radius of droplet still in dispersion at time t, cm.

r_o : Characteristic droplet radius, cm.

Re_o : Modified Reynold's number for multi-particle system ,

Re_η: Reynold's number for a single sphere.

Re_α: Modified Reynold's number for multi-particle system.

Re_ϕ : Modified Reynold's number for multi-particle system.

t: lapsed time for batch settler, s.

u_{bs}: Rates of movement of sedimentation boundary in batch dispersion, cm/s.

u_s: Relative velocity of drops in dispersion, cm/s.

u_{sh}: Terminal velocity of single particle, cm/sec.

$x_{o,}\ x$: Initial and variable position of sedimenting interface relative to final undisturbed interface in batch dispersion, cm.

Greek Letters

β : Size distribution parameter.

τ : Index.

Γ : Relative variance.

ε : Average hold – up.

ε_s :Average hold-up in sedimentation zone.

λ, υ :Coefficient .

μ_c, ,μ_d:Viscosity of continuous and dispersed phases, g/(cm. s).

μ_ε,μ_α, μ_ϕ: Different mean viscosity's of dispersion, g/(cm. s).

γ : Absolute percentage deviation.

ξ : Parameter defined distribution.

ρ_c, ρ_d : Density of continuous and dispersed phases, g/cm^2.

$\Delta\rho$: Density difference between phases (ρ_c -ρ_d).

σ : Interfacial Tension, gm /cm^2.

δ: Coefficient.

τ:Index.

ϕ_{32} : Sauter mean drop diameter, cm.

ϕ_m : Maximum stable drop diameter in mixer, cm.

Ω: Symmetry coefficient.

ω : Parameter defined distribution.

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